

## OPTIMAL CONTROL OF COMPLEX TECHNICAL OBJECTS BASED ON THE PREDICTIVE MODEL

*Annotation:* Optimal methods of control of complex technical objects using a predictive model are considered. A structural and functional scheme for the implementation of automatic optimal stabilization of a given motion of autonomous robotic underwater based on the original methods of parametric identification of predictive models and modal synthesis of optimal regulators on separate forecast horizons is proposed.

*Keyword:* optimal control, predictive model, parametric identification, ACOR, prediction horizon, modal synthesis

### Introduction

Optimization of processes by classical methods and, in particular, the construction of optimal systems, despite the developed theory, encounters difficulties in the implementation of control systems (solution of boundary value problems on the principle of maximum, to reproduce the surface or hypersurface switching, to solve transcendent equations, etc.). However, the acceleration of management processes can be achieved in another way, namely by using the forecasting method. This method began to develop in the 60-ies for the control of processes and equipment in the petrochemical and energy production, for which the use of traditional methods of synthesis was extremely difficult due to the exceptional complexity of their mathematical models. A characteristic feature of this approach is to consistently determine the final result of the impact of the control chosen at the moment.

The method of forecasting was first used in the work of J. Coles and A. Noton [1], further developed X. Chestnut, W. Sollecito and P. Trutmann [2], for optimal control of second order objects with time-varying switching lines. A characteristic feature of these works is the prediction in computer systems with the repetition of the solution of the set of optimal trajectories of the object with the allocation of the trajectory that corresponds to a given final position of the object, as well as the use of logic designed for no more than one switching control action.

In [3] the forecasting method was used to determine the control action by solving the equations of the object dynamics and the Euler variational equation for it in an accelerated time scale. In this case, from the set of integral curves, the one that corresponded to the desired final state of the object was chosen. Features of application of a forecasting method in the mentioned works do not allow to extend them on objects above the second order and on the objects having additional restrictions on coordinates.

To eliminate this drawback in the work [4], a method of optimal or close to optimal control of one class of objects based on predictive devices operating in an accelerated time scale is proposed. The essence of this method of control with forecasting is that the control action is formed on the basis of the results of forecasting the future behavior of the control object. Depending on the assessment of the discrepancy between the final state obtained as a result of such a decision and the specified state. The control is determined at the current time. Therefore, forecasting requires, on the one hand, knowledge of the mathematical description of the dynamics of the object and its current state, and on the other hand, solving the problem of determining the final state of the object with a certain control action. Forecasting in the sense of extrapolation of system dynamics is carried out implicitly in any control system, where, in addition to the mismatch signal, derived control is used.

Since such extrapolation is usually approximate, it does not allow to accurately estimate the final result of the control and determine the optimal solution to the problem. Meanwhile, the creation of computing systems operating in an accelerated time scale and with the repetition of the solution process allowed to obtain qualitatively new results in solving forecasting problems [5]. These computing systems, solving at a fast pace the equations of the object dynamics with the initial conditions corresponding to its current state, at a certain control action, allow us to trace the entire control process up to the final state of the object. It gives an opportunity to choose the best in a certain sense the control action, at least from that class, which can be seen in the prediction.

On the other hand, knowledge of the final results of management allows to avoid exceeding the coordinates of the object of certain boundaries, which established for one reason or another. Together, these factors enable prediction of computers greatly enhanced the process management without risk of the transition of the object into an invalid state.

With the advent of advanced mathematical software for modern computer systems, new approaches in the use of predictive models for optimal control of dynamic objects.

#### Optimization of complex technical objects by MPC-approach

One of the modern formalized approaches to the analysis and synthesis of control systems based on mathematical methods of optimization is the theory of control of dynamic objects using Model Predictive Control (MPC) [6].

The idea of optimizing the predicted movement, which is the basis of the MPC approach, arose within the framework of two independent, but essentially similar approaches. The first of them, called Dynamics Matrix Control (DMC), was developed by the efforts of Shell Oil specialists in the mid-60s [7], and the second – Model Algorithmic Control (MAC) – was developed by French engineers of the chemical industry in the late 60s [8]. On the basis of the latter approach, a commercial software package IDCUM (Identification and Command) was created for the first time, which to a certain extent served as a prototype of modern software support for predictive control methods.

The package of application programs Model Predictive Control Toolbox (MPC Tools)[9] is a set of tools for research and design of control algorithms in

discrete and continuous systems based on predictions of the dynamics of their behavior. The package includes more than 50 specialized functions for the design, analysis and simulation of dynamic systems using predictive control.

At the same time, the authors of the package, taking into account its purpose for the initial development of the ideology of the MPC approach, included in the working tools only those tools that are quite easy to learn and practical application. Model Predictive Control Toolbox is a package for research and design of control algorithms with dynamics prediction. Allows you to create adaptive control systems for complex systems with one or more inputs (outputs) and various restrictions. The package allows you to implement a control principle in which the input impact is calculated at each step based on the internal model of the object. Quadratic programming is used to optimize control.

Currently, the MPC-approach is in the stage of intensive development, as evidenced by the extensive bibliography of scientific works published in recent years on this issue. The development of control ideas with forecasting occurs in the direction of the use of nonlinear models, ensuring the Lyapunov stability of controlled movements, giving robust properties to the closed control system, the use of modern optimization methods in real time, etc. [10,11]. Moreover, the scope of practical application of the MPC approach has expanded significantly, covering a variety of processes in the chemical and construction industry, light and food industries, in aerospace research, in modern energy systems, etc.

The main advantage of the MPC approach, which determines its successful use in the practice of construction and operation of control systems, is the relative simplicity of the basic scheme of formation of the feedback, combined with high adaptive properties. The latter circumstance makes it possible to control multidimensional and multi-connected objects with a complex structure, including nonlinearities, to optimize processes in real time within the constraints on the control and controlled variables, to take into account uncertainties in the assignment of objects and perturbations. In addition, it is possible to take into account the transport delay, taking into account changes in the quality criteria during the process and sensor failures of the measurement system.

The essence of the MPC approach is the following scheme of control of dynamic objects on the principle of feedback. The scheme can be combined with the preliminary identification of the equations of the model used to perform the forecast (Fig. 1).

The implementation of this approach is as follows:

We consider some (relatively simple) mathematical model of the object, the initial conditions for which is its current state. With a given program control, the equations of this model are integrated, which gives a forecast of the object movement on a certain finite period of time (forecast horizon).

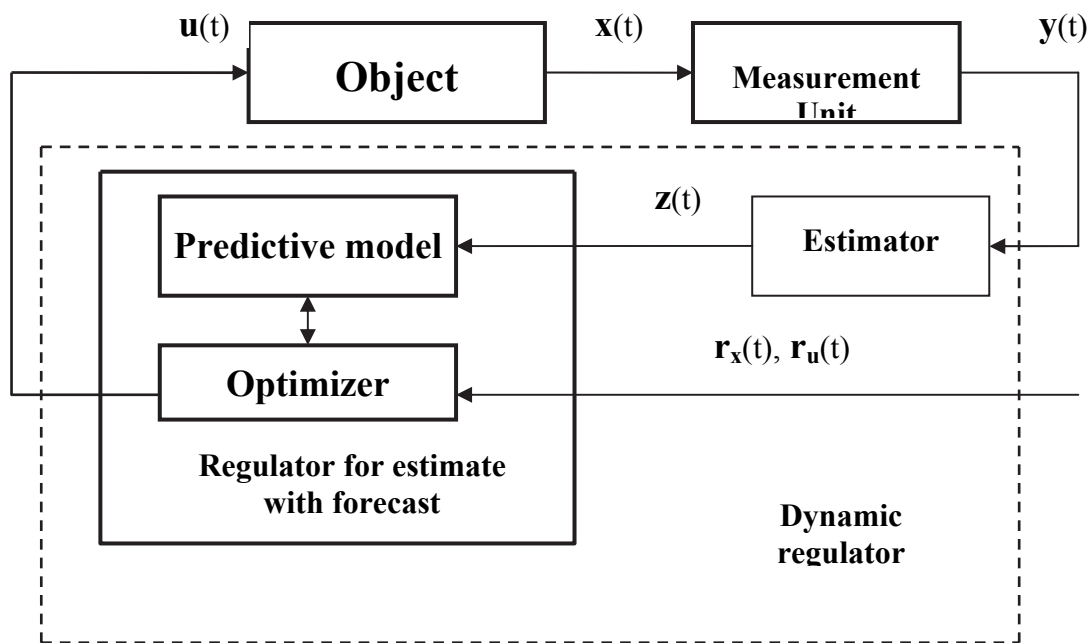


Fig.1. Scheme of implementation of the MPC- approach to the synthesis of optimal control systems with predictive models

After that, the program of optimization of object motion control is developed, the purpose of which is the approximation of the controlled variables of the forecast model to the corresponding given signals on the forecast horizon. Optimization is carried out taking into account the whole complex of restrictions imposed on the control and adjustable variables.

At the calculation step, which is a fixed small part of the forecast horizon, the found optimal control is realized and the actual state of the object is measured (or restored by the measured variables) at the end of the step.

The forecast horizon moves one step forward, and points 1 - 3 of this sequence of actions are repeated.

Optimal stabilization of a given motion of an Autonomous underwater robot

It is known that all real Autonomous robotic underwater vehicles (ARPA) to some extent are nonlinear and non-stationary. One of the urgent tasks in the management of the ARPA is to stabilize the software (given) motion. Analysis and synthesis of systems of optimal stabilization of ARPA program motion, in General, is a rather complex mathematical problem. However, since the majority of ARPA allows you to take as a mathematical model of their dynamics in the stabilization modes of the linearized system of equations, it allows to apply time-vity mathematical apparatus for the solution of linear stationary and non-stationary differential equations to the solution of problems of management of the ARPA. Despite this, the synthesis of optimal stabilization systems for ARPA remains a challenge.

In General, linear non-stationary model of dynamics of the ARPA in the modes of stabilization can be represented in the form:

$$\dot{\bar{x}}(t) = A(t)\bar{x}(t) + B(t)\bar{u}(t), \quad t \in [t_0, T_f], \quad \bar{x}(t_0) = \bar{x}^{(0)}, \quad (1)$$

где  $A(t) = \{a_{ij}(t)\}$ ,  $B(t) = \{b_{ik}(t)\}$  – dimension  $(n \times n)$  and  $(n \times m)$  matrices, respectively, whose coefficients have a constant sign

$$\text{sign}[a_{ij}(t)] = \text{const}, \text{sign}[b_{ik}(t)] = \text{const}, \quad (2)$$

and monotonous

$$\text{sign}[da_{ij}(t)/dt] = \text{const}, \text{sign}[db_{ik}(t)/dt] = \text{const} \quad (3)$$

and there are pre-unknown functions that have continuous first derivatives and are bounded in the domain of determination on the stabilization interval. Linear stationary and quasi-stationary dynamic models will be special cases of the model (1). We also assume that the control vector in solving the problem of stabilization of the ARPA program motion is not limited

The problem of optimal stabilization of program motion the ARPA is formulated as follows: find the control that transforms the system (1) under conditions (2) and (3) from an arbitrary initial state to zero and minimizes the functional:

$$I = \int_{t_0}^{t_k} [\bar{x}^T(t)Q\bar{x}(t) + \bar{u}^T(t)R\bar{u}(t)] dt, \quad (4)$$

where  $t_k$  – fixed,  $Q$  и  $R$  – positive definite matrixes of size  $(n \times n)$ .

This formulation of the problem with functional (4) is known in the literature as the linear-quadratic optimization problem.

### The solution of the problem

For accurate realization of ARPA movement along a given path, an automated optimal stabilization of the programmed motion is required. We consider that on ARPA the onboard control system (BSC) with the corresponding mathematical and software is established. On the basis of such BSC the structural and functional scheme of automation of the process of optimal stabilization of ARPA motion can be realized (Fig.2.).

Marked: BFPT – block of forming unit programmed trajectory; CMD – complete model of the dynamics of the ARPA; BUC – block of unit calculation program management; BSP1, BSP2 – blocks of sensors of parameters; BIP – block identifying the parameters of forecasting models lactation; BSSL – block for the synthesis of stabilization laws.

According to this structural and functional scheme, the automation of the process of optimal stabilization of the motion ARPA is carried out as follows. By the time of performance of some planned mission to the input of the BSC, a given program trajectory of ARPA motion receive in the form of a discrete series of spatial coordinates (or in analytical form) in certain areas of motion. In the first case, the BFPT block approximates a discrete series of coordinates by an analytical representation, in particular, pro-

posed by the authors in [12] spline approximation. In the second case, the need for this disappears. Software trajectory through CMD is included in the BUC and via BS1 is supplied to ARPA. In addition, for certain areas of the software movement the ARPA on the basis of the information management program and a program path in BIP may be construct corresponding plots of the linearized model of the dynamics of ARPA, and evaluation of its parameters according to the work of the authors [13] in the form of a series of Walsh using the algorithm of the adaptive partitioning of the time interval of movement. The adaptive algorithm, in contrast to the fixed partitioning, allows to approximate the original ARPA model at individual sites with a given accuracy in the form of quasi-stationary predictive models.

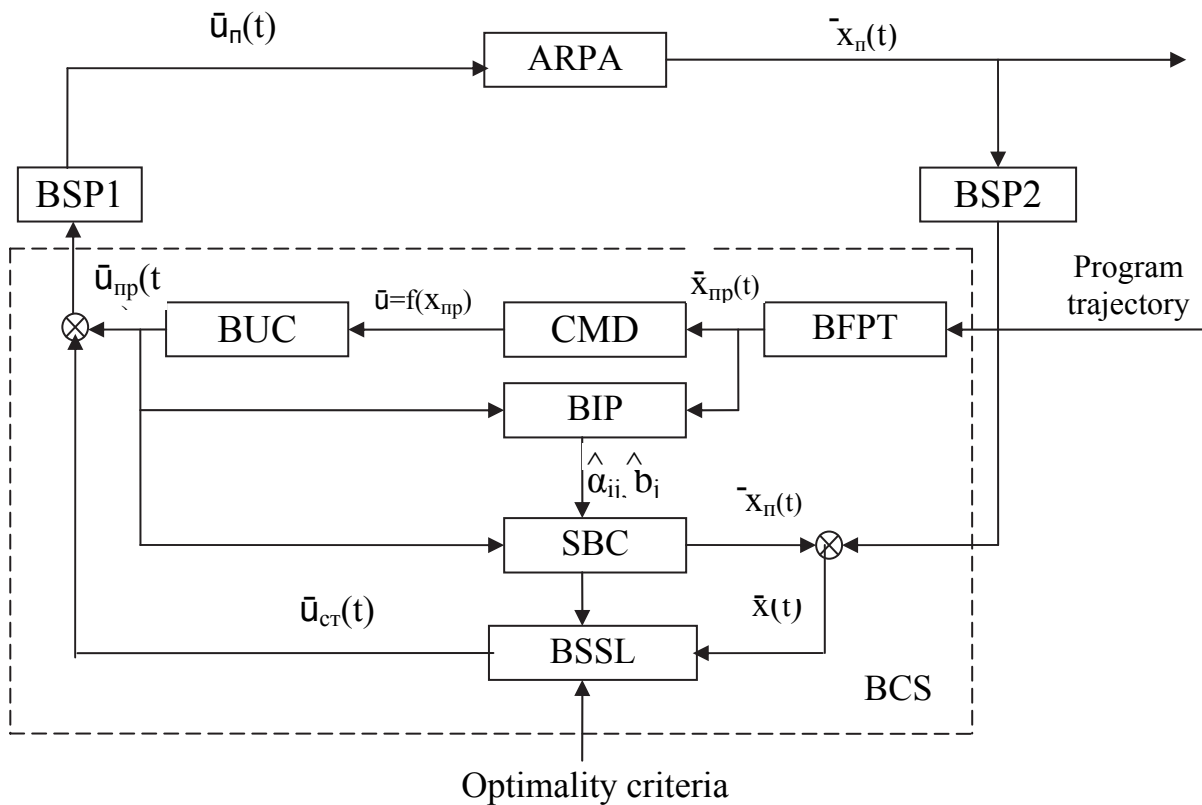


Fig.2. Structural and functional scheme of automation of the process of optimal stabilization of motion ARPA

As an example, the problem of parametric identification of ARPA with the dynamics of the second order is considered

$$\begin{aligned} \dot{x}_1(t) &= a_{12}(t)x_2(t), \\ \dot{x}_2(t) &= b_2(t)u(t), \quad t \in [0,100], \end{aligned} \tag{5}$$

where  $x_1(0) = 30, x_2(0) = 50, b_2(t) = 1, u(t) = -1$ . Let the exact value of the estimated parameter be  $a_{12}(t) = 0,000012t^3 - 0,0014t^2 + 0,033t + 2$ . Parameters for comparison of constant and adaptive algorithms were set as follows:  $L = 10; P = 5; \delta' = 1; \delta_l = 1$  для всех  $l; \eta_0 = 10; \varepsilon = 0,2; \mu = 0,5$ . The results of the

evaluation  $\hat{a}_{12}(t)$  for a fixed ( $L = 10$ ) and adaptive partitioning of the interval are given in Fig.3, where curve 1 is the exact value  $a_{12}(t)$ ; curve 2 – evaluation  $\hat{a}_{12}(t)$  for a fixed partition; curve 3 – evaluation  $\hat{a}_{12}(t)$  at adaptive partitioning. The accuracy of the parameter estimation  $a_{12}(t)$  is characterized by the value of

$$\delta^2 = \sum_{m=0}^M [\delta \hat{a}_{12}(t_m)]^2 / \sum_{m=0}^M [a_{12}(t_m)]^2.$$

Comparison of the estimates obtained by the fixed and adaptive algorithms for the considered system (5) allows us to conclude that the parameter estimation accuracy can be increased using an algorithm with adaptive selection of the quasi-stationarity interval.

Further, from the analysis of the approximation of all parameters of the quasi-stationary model by piecewise constant Walsh functions, we determine the minimum partition interval, which is taken as the forecast horizon, since all parameters of the predictive models are simultaneously constant on it. Thus, we form stationary predictive models with the selected forecast horizon.

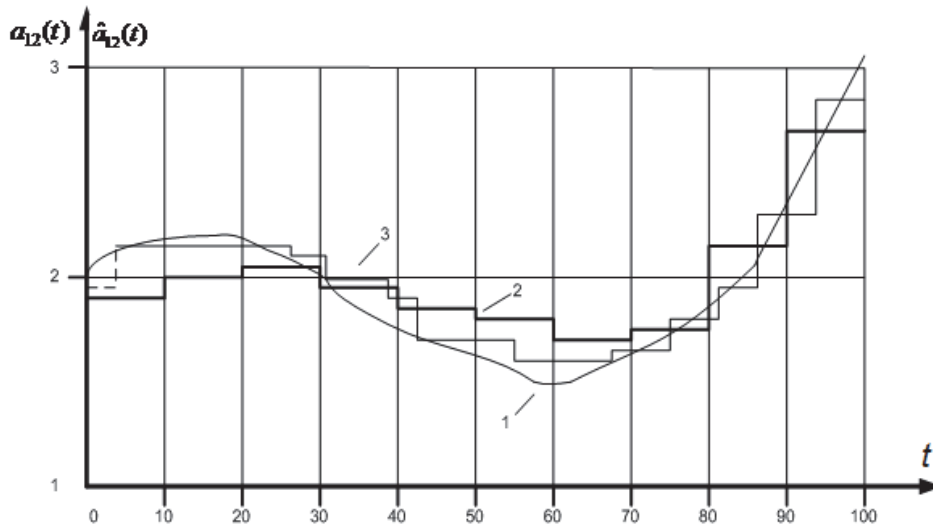


Fig. 3. The results of estimation of parameter  $a_{12}(t)$  with a fixed and adaptive partitioning of the time interval

These data are used in the obtained quasi-stationary models of CMD ARPA dynamics, at the output of which a given trajectory is formed and compared with the current trajectory coming through BS2. The appearance of inconsistency between the program and the current trajectories is included in the block BSSL, which, taking into account the optimality criteria set from the outside, automatically generates in real time the optimal stabilization law for this interval. In this case, the problem of ACOR can be successfully solved in real time, proposed in the authors' work by the method of modal synthesis based on the method of uncertain coefficients [14], which allows to provide the specified indicators of transients.

## Summary

The solution of optimal control problems with predictive models is one of the modern methods of research of control systems and is easily implemented with the help of such systems as Matlab, which with its packages has great opportunities and tools for solving problems of analysis, synthesis and simulation of such control systems. A sequential approach to parametric identification of predictive ARPA models and modal synthesis of optimal regulators based on the method of uncertain coefficients allows for automatic optimal stabilization of ARPA in real time.

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