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FINDING A COMPROMISE SOLUTION TO THE TRANSPORTATION PROBLEM UNDER UNCERTAINTY

Abstract: The paper considers the solution of one combinatorial optimization problem under uncertainty. Its functional in deterministic formulation is a linear convolution of weights and arbitrary characteristics of a feasible solution. A constructive algorithm for its solving is a general linear programming problem solution algorithm. Under the uncertainty we mean the ambiguity of the weight values in the functional we optimize. We propose a new approach for finding a compromise solution by the criterion of minimizing the total weighted excess of "desired" upper bounds on the optimal values of particular functionals. The basis of this approach is the construction and solution of some linear programming problem. We illustrate this approach on a transportation problem example in conditions of uncertainty.

Key words: Uncertainty, combinatorial optimization, compromise solution, transportation problem of linear programming.

Introduction

Various decisions have to be regularly made in business practice. An important condition for rational decision making is to have as accurate information as possible about the decision subject and its consequences. However, obtaining accurate and complete information on which decision making is based is often not possible. Uncertainty arises when it is not possible to assess the future development of events both in terms of their fulfillment probability and from the point of view of their manifestation type.

Papers [1, 2] introduced a new class of combinatorial optimization problems under uncertainty with the following properties:

1) the optimization criterion is a linear convolution of weights and arbitrary numerical characteristics of a feasible solution;

2) there exists an efficient algorithm for the problem solving in a deterministic formulation and any structural change in its range of feasible solutions (for example, adding a linear constraint) makes the algorithm's application impossible;

3) the uncertainty in the combinatorial optimization problem solution refers to the ambiguity of the weight coefficients' values included in the optimization criterion.

Authors of many publications use a linear convolution for problems solving under uncertainty. However, all works are devoted to solving multi-criteria problems from different areas of economics and technology by reducing them to single-criterion problems (e.g., [3–7]). We have not found the use of a linear convolution of weight coefficients and arbitrary numerical characteristics of a feasible solution for an efficient solving of problems of the abovementioned class under uncertainty. For example, the method of finding a compromise solution to the two-criteria linear programming problem in [8] uses the Lp-metric. However, our approach is fundamentally different already in that we need to solve only one, not three, optimization problems to find a compromise solution. The problem in our statement is considered for the first time: there is no definition of the abovementioned sufficiently wide class of combinatorial optimization problems in papers (e.g., [9–11]). And complex and time-consuming methods of stochastic and robust optimization are developed to solve various forms of uncertainty. However, most of the publications reviewed highlighted the critical importance of taking into account dynamism and uncertainty, in particular when planning and scheduling in practice, and the need to consider this uncertainty in decision models. Therefore, combinatorial optimization problems under uncertainty attract considerable attention from both practitioners and academia [12]. A comprehensive literature survey of aggregate production planning under uncertainty given in [13] concluded that a large portion of the existing research studies the deterministic state of the planning and ignores its inherent uncertain nature. In practice, this may lead to considerable errors and imprecise decisions.

Papers [1, 2] describe the theoretical basics for solving the considered class of problems. The authors propose there five compromise criteria for the uncertainty resolving. Also, they identify new properties of linear convolution that form the basis of efficient formal procedures for compromise solutions finding. Paper [14] illustrates by specific examples the efficiency of these theoretical propositions for single product and multiple products transportation problems. The paper also substantiates the extension of the class of combinatorial optimization problems under uncertainty introduced in [1, 2].

General theoretical positions

The main provisions of the theory described in [1, 2, 15] are as follows. We study the class of combinatorial optimization problems of the following form:

$$\min_{\sigma\in\Omega} \sum_{i=1}^{s} \omega_i k_i(\sigma) \tag{1}$$

where ω_i are numbers, $k_i(\sigma)$ is i-th arbitrary numerical characteristic of a feasible solution σ $i=\overline{1,s}$, Ω is the set of all feasible solutions.

There are R sets of weights $\{\omega_i^r,\ i=\overline{1,s}\},\ r=\overline{1,R}$. Each one may be a set of coefficients $\omega_1,\dots,\omega_s$ of the problem (1) at the stage of fulfillment of its solution. We need to find a feasible solution $\sigma\in\Omega$ that satisfies one of the below mentioned criteria.

Let us denote:

$$f_{opt}^r=\min_{\sigma\in\Omega}\sum_{i=1}^{s}\omega_i^r k_i(\sigma),\ \{\sigma_r\}=\arg\min_{\sigma\in\Omega}\sum_{i=1}^{s}\omega_i^r k_i(\sigma),$$

$$L_r=\sum_{\substack{m=1\\ m\neq r}}^{R}\left(\sum_{i=1}^{s}\omega_i^m k_i(\sigma_r)-f_{opt}^m\right).$$

Remark 1. If $\{\sigma_r\}$ consists of more than one solution, we keep the one on which we have $\min_{\{\sigma_r\}} L_r$ and denote this solution by σ_r (we show in [1] how to obtain σ_r for the case when Ω is finite).

Suppose that $L_p=\min_r L_r$ (L_p corresponds to a solution σ_p).

Criterion 1. We need to find σ that reaches

$$\min_{\sigma\in\Omega}\sum_{r=1}^{R}\left(\sum_{i=1}^{s}\omega_i^r k_i(\sigma)-f_{opt}^r\right). \qquad (2)$$

Criterion 2. Find a feasible solution that satisfies the condition

$$\min_{\sigma\in\Omega}\sum_{r=1}^{R}a_r\left(\sum_{i=1}^{s}\omega_i^r k_i(\sigma)-f_{opt}^r\right) \qquad (3)$$

where $a_r>0,\ r=\overline{1,R}$ are the coefficients set by experts.

Criterion 3. Let us introduce a random variable $F=\sum_{i=1}^{s}\overline{\omega}_i k_i(\sigma)-\overline{f}_{opt}$ where s+1-dimensional discrete random variable $\overline{\omega}_1\ \dots,\overline{\omega}_s,\ \overline{f}_{opt}$ is specified by the table:

$$\left\{\begin{array}{c}\omega_1^r,\dots,\omega_s^r,\ f_{opt}^r\\ p_r>0,\ r=\overline{1,R}\end{array}\right\}.$$

We need to find a solution to this problem:

$$\min_{\sigma\in\Omega}MF=\min_{\sigma\in\Omega}\sum_{r=1}^{R}p_r\left(\sum_{i=1}^{s}\omega_i^r k_i(\sigma)-f_{opt}^r\right). \qquad (4)$$

Criterion 4. Find a feasible solution $\sigma(\Delta_1,\dots,\Delta_R)$ for which

$$\Delta_i\le l_i,\ l_i>0,\ i=\overline{1,R}. \qquad (5)$$

Here, $\sigma(\Delta_1, \ldots, \Delta_R)$ is a feasible solution $\sigma \in \Omega$ that has the specified in brackets deviations from the optimums for each set of weights:

$$\Delta_r = \sum_{i=1}^{s} \omega_i^r k_i(\sigma) - f_{opt}^r, \ r = \overline{1, R}.$$

Criterion 5. For one of the sets of weights ω_i^r, $i = \overline{1, s}$, $r \in \{\overline{1, R}\}$, find an optimal solution corresponding to $\min\limits_{\sigma \in \{\sigma_r\}} \sum\limits_{j=1, j \neq r}^{R} \Delta_j$.

For the case of impossibility of finding a solution that satisfies criterion 4, papers [1, 2] propose a formal procedure for a compromise solution finding. The procedure is based on the use of expert constraint weights $\Delta_i \le l_i$, $i = \overline{1, R}$.

In this paper, in contrast to [14], we propose a new approach to find a compromise solution according to criterion 4 for the case when the constructive algorithm for the combinatorial optimization problem solving in deterministic formulation is a linear programming (LP) algorithm. The approach is as follows.

We need to find a feasible solution $\sigma(\Delta_1, \ldots, \Delta_R)$ for which

$$\sum_{r=1}^{R} \alpha_r y_r \tag{6}$$

reaches minimum. Here, α_r, $r = \overline{1, R}$, are weight coefficients; y_r, $r = \overline{1, R}$, are the violations in the corresponding inequalities in the system (5). Their values are determined by the following restrictions:

$$\Delta_r - y_r = \sum_{i=1}^{s} \omega_i^r k_i(\sigma) - f_{opt}^r - y_r \le l_r, \ r = \overline{1, R}, \tag{7}$$

$$y_1, \ldots, y_R \ge 0. \tag{8}$$

If $\sum\limits_{r=1}^{R} y_r = 0$, then the obtained solution satisfies the constraints (5) and is optimal by criterion 4. If $\sum\limits_{r=1}^{R} y_r > 0$, then the obtained compromise solution is optimal by criterion (6).

Finding a transportation problem solution in uncertainty conditions

Let us illustrate the proposed approach to a transportation problem under uncertainty in the following form:

$$f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \to \min, \tag{9}$$

$$C = \{c_{ij}\}_{j=\overline{1,n}}^{i=\overline{1,m}} = \begin{cases} C^1 = \{c_{ij}^1\}_{j=\overline{1,n}}^{i=\overline{1,m}} \\ or \\ C^2 = \{c_{ij}^2\}_{j=\overline{1,n}}^{i=\overline{1,m}} \\ or \\ \dots \\ C^R = \{c_{ij}^R\}_{j=\overline{1,n}}^{i=\overline{1,m}} \end{cases}, \qquad (10)$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = \overline{1,m}, \qquad (11)$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = \overline{1,n}, \qquad (12)$$

$$x_{ij} \geq 0,\ i = \overline{1,m};\ j = \overline{1,n}. \qquad (13)$$

Problem (9)–(13) is a transportation problem for which the transportation costs matrix can take one of R values $C^r = \{c_{ij}^r\}_{j=\overline{1,n}}^{i=\overline{1,m}}$, $r = \overline{1,R}$, and we do not know in advance which of these matrices will be relevant in time of the solution's fulfillment.

Let us match the notation of the general theory to the notation of the problem (9)–(13):

– the set Ω is given by constraints (11)–(13);

– x is a feasible solution, $x = \{x_{ij}\}_{j=\overline{1,n}}^{i=\overline{1,m}}$ (analog of σ);

– x_{ij} is (ij)-th numerical parameter of a feasible solution x;

– the function $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ is the function $\sum_{i=1}^{s} \omega_i k_i(\sigma)$ in the conditions of the problem we consider.

The problem (9)–(13) has all the properties of the studied class of combinatorial optimization problems under uncertainty. It can be solved by any known method, for example, by the potential method or by the Hungarian algorithm.

Let us call the objective function $f = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij} \to min$ a particular criterion. Let TPr denote a (particular) transportation problem with the objective function $f = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij} \to min$ and constraints (11)–(13), x^r an optimal solution to this problem ($r = \overline{1,R}$).

In order to find a solution to the problem (9)–(13) that satisfies criterion 4, we need to find a solution to the following LP problem:

$$\sum_{r=1}^{R} \alpha_r y_r \to min, \tag{14}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij}^r - y_r \le l_r,\ r = \overline{1,R}, \tag{15}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = \overline{1,m}, \tag{16}$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = \overline{1,n}, \tag{17}$$

$$x_{ij} \ge 0,\ i = \overline{1,m};\ j = \overline{1,n}, \tag{18}$$

$$y_r \ge 0,\ r = \overline{1,R}. \tag{19}$$

Putting into (15) the optimal values of the objective functions TPr, $r = \overline{1,R}$, we obtain:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij} - f_{opt}^r - y_r \le l_r.$$

Let us apply the proposed approach of finding a compromise solution of the transportation problem's under uncertainty to solve instances with the dimension: $m = 7$, $n = 6$.

Example 1. Given: $R = 2$; $\alpha_1 = \alpha_2 = 1$;

$$a = \begin{bmatrix} 20 \\ 25 \\ 30 \\ 40 \\ 10 \\ 15 \\ 33 \end{bmatrix},\ b = \begin{bmatrix} 14 \\ 41 \\ 27 \\ 22 \\ 31 \\ 38 \end{bmatrix},\ C^1 = \begin{bmatrix} 5 & 2 & 5 & 3 & 9 & 2 \\ 3 & 4 & 2 & 5 & 2 & 8 \\ 3 & 4 & 5 & 3 & 7 & 5 \\ 8 & 5 & 9 & 9 & 1 & 4 \\ 4 & 7 & 8 & 2 & 9 & 9 \\ 2 & 4 & 6 & 5 & 3 & 2 \\ 2 & 8 & 7 & 9 & 6 & 6 \end{bmatrix},\ C^2 = \begin{bmatrix} 7 & 6 & 3 & 3 & 4 & 5 \\ 2 & 8 & 6 & 8 & 3 & 8 \\ 4 & 2 & 3 & 6 & 4 & 6 \\ 3 & 5 & 3 & 5 & 9 & 7 \\ 4 & 7 & 3 & 6 & 3 & 3 \\ 5 & 2 & 2 & 6 & 6 & 5 \\ 7 & 4 & 4 & 5 & 4 & 6 \end{bmatrix}. \tag{20}$$

We need to find a solution for which

$$\Delta_1 \le l_1,\ \Delta_2 \le l_2,\ l_1 > 0,\ l_2 > 0.$$

We give in Table 1 the optimal solutions of TP1 and TP2, as well as the corresponding values of particular criteria.

Table 1

Problem	Optimal solution	Criterion value	Δ_1	Δ_2
TP1	$x^1 = \begin{bmatrix} 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 & 0 \\ 0 & 18 & 0 & 12 & 0 & 0 \\ 0 & 3 & 0 & 0 & 31 & 6 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \\ 14 & 0 & 2 & 0 & 0 & 17 \end{bmatrix}$	$f^1_{opt}=462$	0	489
TP2	$x^2 = \begin{bmatrix} 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 \\ 14 & 0 & 26 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 11 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 6 & 25 \end{bmatrix}$	$f^2_{opt}=568$	464	0

Example 1.1. Let the following restrictions be imposed on the quantities of Δ_1 and Δ_2:

$$\Delta_1 \le 140, \tag{21}$$

$$\Delta_2 \le 120. \tag{22}$$

The solutions x^1 and x^2 do not satisfy the constraints (21)–(22). In this case, according to the proposed approach, we need to solve the following LP problem:

$$y_1 + y_2 \to min, \tag{23}$$

$$\sum_{i=1}^{7}\sum_{j=1}^{6} c^1_{ij}x_{ij} - 462 - y_1 \le 140, \tag{24}$$

$$\sum_{i=1}^{7}\sum_{j=1}^{6} c^2_{ij}x_{ij} - 568 - y_2 \le 120, \tag{25}$$

$$\sum_{j=1}^{6} x_{ij} = a_i,\ i = \overline{1,\ 7}, \tag{26}$$

$$\sum_{i=1}^{7} x_{ij} = b_j,\ j = \overline{1,\ 6}, \tag{27}$$

$$x_{ij} \ge 0,\ i = \overline{1,\ 7};\ j = \overline{1,\ 6}. \tag{28}$$

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$$y_1, y_2 \geq 0 . \tag{29}$$

After solving this problem, we obtain the integer solution:

$$\overline{x} = \begin{bmatrix} 0 & 0 & 0 & 12 & 0 & 8 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 22 & 8 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 6 & 30 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 15 & 0 & 0 & 0 & 0 \\ 14 & 0 & 19 & 0 & 0 & 0 \end{bmatrix}.$$

In this solution, $f^1(\overline{x}) = 617$, $\Delta_1 = 155$, $y_1 = 15$, $f^2(\overline{x}) = 767$, $\Delta_2 = 199$, $y_2 = 79$. Total excess of "desired" upper bounds on the values of Δ_1 and Δ_2 is $y_1 + y_2 = 94$. The solution $\overline{x}$ is an optimal compromise solution by criterion (6).

Example 1.2. Let $\Delta_1 \leq 270$, $\Delta_2 \leq 170$. After solving the corresponding LP problem, we obtain the integer solution:

$$\overline{x} = \begin{bmatrix} 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 2 & 0 & 23 & 0 \\ 4 & 1 & 25 & 0 & 0 & 0 \\ 0 & 40 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 13 \\ 0 & 0 & 0 & 0 & 8 & 25 \end{bmatrix}.$$

Here, $\Delta_1 = 263$, $y_1 = 0$, $\Delta_2 = 165$, $y_2 = 0$, $y_1 + y_2 = 0$. The obtained solution is optimal by criterion 4.

Example 1.3. Let $\Delta_1 \leq 150$, $\Delta_2 \leq 150$. Having solved the corresponding LP problem, we obtain the non-integer solution:

$$\overline{x} = \begin{bmatrix} 0 & 0 & 0 & 13.67 & 0 & 6.33 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 23.67 & 6.33 & 0 & 0 & 0 \\ 0 & 17.33 & 0 & 0 & 6 & 16.67 \\ 1.67 & 0 & 0 & 8.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \\ 12.33 & 0 & 20.67 & 0 & 0 & 0 \end{bmatrix}.$$

In it, $f^1(\overline{x}) = 612$, $\Delta_1 = 150$, $y_1 = 0$, $f^2(\overline{x}) = 772$, $\Delta_2 = 204$, $y_2 = 54$, $y_1 + y_2 = 54$. In this case, we need either to round the solution (if rounding leads to a feasible solution) or to use the iterative procedure for a compromise solution finding described in [1, 2]. We presented its application for an individual problem solving in [14].

The rounded solution is this:

$$\overline{x} = \begin{bmatrix} 0 & 0 & 0 & 14 & 0 & 6 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 24 & 6 & 0 & 0 & 0 \\ 0 & 17 & 0 & 0 & 6 & 17 \\ 2 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \\ 12 & 0 & 21 & 0 & 0 & 0 \end{bmatrix}.$$

It is feasible, its characteristics are: $f^1(\overline{x}) = 614$, $\Delta_1 = 152$, $y_1 = 2$, $f^2(\overline{x}) = 770$, $\Delta_2 = 202$, $y_2 = 52$, $y_1 + y_2 = 54$. This solution is an optimal compromise solution by criterion (6).

Example 2. $R = 4$. We defined vectors a, b, matrices C^1 and C^2 in (20). Matrices C^3 and C^4 are as follows:

$$C^3 = \begin{bmatrix} 3 & 7 & 8 & 8 & 2 & 2 \\ 5 & 2 & 4 & 6 & 4 & 8 \\ 2 & 9 & 2 & 3 & 7 & 6 \\ 4 & 6 & 5 & 7 & 3 & 7 \\ 4 & 6 & 5 & 4 & 5 & 2 \\ 4 & 9 & 5 & 2 & 5 & 3 \\ 4 & 2 & 5 & 4 & 9 & 4 \end{bmatrix}, C^4 = \begin{bmatrix} 2 & 5 & 8 & 6 & 6 & 9 \\ 5 & 5 & 5 & 4 & 5 & 9 \\ 9 & 8 & 8 & 5 & 7 & 8 \\ 4 & 2 & 5 & 9 & 7 & 8 \\ 6 & 6 & 9 & 6 & 6 & 5 \\ 9 & 7 & 4 & 2 & 3 & 6 \\ 8 & 8 & 7 & 9 & 3 & 6 \end{bmatrix}.$$

The corresponding to them optimal solutions and the values of particular criteria are:

$$x^3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 20 \\ 0 & 25 & 0 & 0 & 0 & 0 \\ 3 & 0 & 27 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 31 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 15 & 0 & 0 \\ 2 & 16 & 0 & 7 & 0 & 8 \end{bmatrix}, f^3_{opt} = 429,$$

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$$x^4 = \begin{bmatrix} 14 & 3 & 0 & 3 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 26 \\ 0 & 38 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 31 & 2 \end{bmatrix}, \; f_{opt}^4 = 685 .$$

We give in Table 2 the values of Δ_r, $r = \overline{1, 4}$, for the obtained solutions.

Table 2

Solution	Δ_1	Δ_2	Δ_3	Δ_4
x^1	0	468	381	393
x^2	464	0	582	321
x^3	333	412	0	451
x^4	386	339	501	0

Example 2.1. Suppose $\alpha_1 = 2.5$, $\alpha_2 = 2$, $\alpha_3 = 1.5$, $\alpha_4 = 1$, and let the restrictions $\Delta_1 \le 100$, $\Delta_2 \le 100$, $\Delta_3 \le 100$, $\Delta_4 \le 100$ be imposed on the values of Δ_r, $r = \overline{1, 4}$. Having solved the corresponding LP problem, we obtain the integer solution:

$$\overline{x} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 20 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 18 & 12 & 0 & 0 \\ 0 & 34 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \\ 14 & 7 & 9 & 0 & 0 & 3 \end{bmatrix}.$$

In it, $\Delta_1 = 145$, $\Delta_2 = 272$, $\Delta_3 = 217$, $\Delta_4 = 333$, $\sum_{r=1}^{4} \alpha_r y_r = 865$. This solution is an optimal compromise solution by criterion (6).

Example 2.2. Suppose $\alpha_1 = 1$, $\alpha_2 = 1.5$, $\alpha_3 = 2$, $\alpha_4 = 2.5$, and let the restrictions $\Delta_1 \le 200$, $\Delta_2 \le 200$, $\Delta_3 \le 200$, $\Delta_4 \le 200$ be imposed on the values of Δ_r, $r = \overline{1, 4}$. Having solved the corresponding LP problem, we obtain the non-integer solution:

$$\overline{x} = \begin{bmatrix} 5.01 & 0 & 0 & 0 & 0 & 14.99 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 27 & 3 & 0 & 0 \\ 8.46 & 31.22 & 0 & 0 & 0.32 & 0 \\ 0 & 0 & 0 & 4 & 0 & 6 \\ 0 & 0 & 0 & 15 & 0 & 0 \\ 0.53 & 9.78 & 0 & 0 & 5.68 & 17.01 \end{bmatrix}.$$

It has $\Delta_1 = 364$, $\Delta_2 = \Delta_3 = \Delta_4 = 200$, $\sum_{r=1}^{4} \alpha_r y_r = 164$. Rounding leads to infeasibility of the solution (the 4th and 7th restrictions in the system (16) are not satisfied). In this case, we need to apply the iterative procedure for a compromise solution finding we described in [14].

Conclusions

1. If the solution of a transportation problem under uncertainty should not be necessarily integer, then the compromise solution is the solution of a single LP problem with the criterion of minimizing the total weighted excess of the “desired” upper bounds on the optimal values of the particular functionals ($\sum_{r=1}^{R} \alpha_r y_r \to min$).

2. If the solution of a transportation problem must be integer, then we can find a compromise solution that satisfies the criterion 4 using the following scheme.

1) solve the LP problem with the criterion of minimizing the total weighted excess of the “desired” upper bounds on the optimal values of the particular functionals;

2) if the resulting solution is integer, then it is the desired compromise solution;

3) if the obtained solution is non-integer, then we round it. If the solution becomes infeasible after rounding (it does not satisfy the transportation problem’s restrictions), then we find the compromise solution according to criterion 4 using the iterative procedure from [14].

REFERENCES

1. Pavlov A.A. Optimization for one class of combinatorial problems under uncertainty. Interdepartmental scientific-technical journal «Adaptive systems of automatic control». 2019. № 1 (34). P. 81–89. DOI: 10.20535/1560-8956.1.2019.178233

2. Pavlov A.A. Combinatorial optimization under uncertainty and formal models of expert estimation. Bulletin of National Technical University "KhPI". Series:

System Analysis, Control and Information Technologies. 2019. № 1. P. 3–7. DOI: 10.20998/2079-0023.2019.01.01

3. Ногин В.Д. Линейная свертка критериев в многокритериальной оптимизации. Искусственный интеллект и принятие решений. 2014. №4. С. 73–82.

4. Gorelik V.A., Zolotova T.V. Problem of selecting an optimal portfolio with a probabilistic risk function. Journal of Mathematical Sciences. 2016. № 216 (5). С. 603–611.

5. Kharchenko A., Halay I., Zagorodna N., Bodnarchuk I. Trade-off optimal decision of the problem of software system architecture choice. 2015 Xth International scientific and technical conference "Computer Sciences and Information Technologies" (CSIT). Lviv, 2015. P. 198–205. DOI: 10.1109/stc-csit.2015.7325465

6. Новикова Н.М., Поспелова И.И., Зенюков А.И. Метод сверток в многокритериальных задачах с неопределенностью. Известия Российской академии наук. Теория и системы управления. 2017. № 5. С. 27–45. DOI: 10.7868/S0002338817050031

7. Bindima T., Elias E. A novel design and implementation technique for low complexity variable digital filters using multi-objective artificial bee colony optimization and a minimal spanning tree approach. Engineering Applications of Artificial Intelligence. 2017. Vol. 59. P. 133–147. DOI: 10.1016/j.engappai.2016.12.011

8. Mirzapour S.M.J., Hashem Al.-E., Malekly H., Aryanezhad M.B. A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty. International Journal of Production Economics. 2011. Vol. 134 (1). P. 28–42. DOI: 10.1016/j.ijpe.2011.01.027

9. Ємець О., Барболіна Т.М. Лінійні оптимізаційні задачі на розміщеннях з імовірнісною невизначеністю: властивості і розв'язання. Системні дослідження та інформаційні технології. 2016. № 1. С. 107–119. DOI: 10.20535/SRIT.2308-8893.2016.1.11

10. Планирование функционирования предприятия в условиях риска и неопределенности во внешней и внутренней среде. Научно-технические ведомости СПбГПУ. Экономические науки. 2017. № 5 (10). С. 172–183.

11. Shang C., You F. Distributionally robust optimization for planning and scheduling under uncertainty. Computers & Chemical Engineering. 2018. Vol. 110. P. 53–68. DOI: 10.1016/j.compchemeng.2017.12.002

12. Shi Y., Haase C. Optimal trade-offs of aggregate production planning with multiple objective and multi-capacity demand levels. International Journal of Operations and Quantitative Management. 1996. Vol. 2 (2). P. 127–143.

13. Jamalnia A., Yang J.-B., Feili A., Xu D.-L. Jamali G. Aggregate production planning under uncertainty: a comprehensive literature survey and future research directions. The International Journal of Advanced Manufacturing Technology. 2019. Vol. 102 (1–4). P. 159–181. DOI: 10.1007/s00170-018-3151-y

14. Павлов А.А., Жданова Е.Г. Транспортная задача в условиях неопределенности. Международный научно-технический журнал "Проблемы управления и информатики". 2020. № 2 (in press).

15. Pavlov A.A. Long-Term Operational Planning of a Small-Series Production Under Uncertainty (Theory and Practice). The Second International Conference on Computer Science, Engineering and Education Applications ICCSEEA 2020. Kiev, 2020 (in press)