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RECOVERY OF HARMONIC SIGNAL FROM DISCRETE SAMPLES

Abstract: Measurement of harmonic signal parameters is based on the use of analog-to-digital conversion (ADC) and microprocessor devices. It is important to have an accurate and stable measurement process. A method for recovery a harmonic signal from discrete samples is proposed. The accuracy of the calculation of signal parameters was evaluated.

Keywords: ADC, amplitude, accuracy rating, frequency, phase.

Description of the problem

It is known [1-3] that the problem of determining the parameters of analog signal from its discrete samples, from the point of view of mathematics, is nothing but the well-known problem of interpolating a continuous function *F(x)* from a finite number *N* of its points X_0, $X_1, \dots X_i, \dots X_N$. The classical problem solving of constructing an interpolation curve is to use the interpolation formula, which is a consequence of the Kotelnikov theorem, in the form of a sum of basis functions of the form *Sin(X)/X*. This approach involves the use of fast Fourier transform algorithms.

As noted in the literature [4-6], the error in determining the amplitude and frequency of a noisy harmonic signal when applying the fast Fourier transform (FFT) method can be 1.5-2%, which may be unacceptable.

Therefore, to determine with high accuracy the parameters of the signals under the action of noise, it is desirable to use methods that have lesser errors.

For an important special case, when there is reason to believe that the signal under consideration has only one spectral component, a simplified approach to determining the amplitude, frequency, and phase of harmonic signal from several ADC samples is considered.

Solution to the problem of recovery a signal from its samples

Problem statement: for several samples of the harmonic signal voltage obtained using the ADC, calculate its unknown parameters: amplitude U_m, frequency $\omega = 2\pi f$ and the initial phase φ_0. It is assumed that the initial phase can be tied to one of the samples.

Such a task is a special case of the problem of recovery an analog signal from its discrete samples. Additional restrictions are imposed that facilitate the solution, namely: it is known that a harmonic signal has only one component in its spectrum. Simplification of the task should allow to have time to calculate the signal parameters for the time between ADC readings. The interval between ADC readings Δt is determined in accordance with the well-known theorem proved by V.A. Kotelnikov in 1933, and should not be more than $\frac{1}{2f}$.

We assume that the measured signal can be represented as

$$U_i = U_m cos(\omega t_i + \varphi_0).$$

Since the number of unknown parameters is three (ω, φ_0, U_m), three equations are needed to find three unknowns:

$$\begin{cases} U_1 = U_m cos(\omega t_1 + \varphi_0) \\ U_2 = U_m cos(\omega t_2 + \varphi_0). \\ U_3 = U_m cos(\omega t_3 + \varphi_0) \end{cases}$$

The initial phase φ_0 must be coincide to one of the samples. Further, we assume that this is the phase at the time of the central second sample.

If ADC codes are used to recovery the signal, which are obtained as a result of digitizing the signal at regular intervals, then it’s convenient to apply in calculations $\mathrm{t}_1 = -\Delta\mathrm{t}$, $\mathrm{t}_2 = 0$, $\mathrm{t}_3 = +\Delta\mathrm{t}$:

$$\begin{cases} U_1 = U_m cos(\omega t_1 + \varphi_0) = U_m cos(\omega t_1) \cdot \cos\varphi_0 - U_m \sin(\omega t_1) \cdot \sin\varphi_0 \\ U_2 = U_m cos(\omega t_2 + \varphi_0) = U_m cos(\omega t_2) \cdot \cos\varphi_0 - U_m \sin(\omega t_2) \cdot \sin\varphi_0 = \\ U_3 = U_m cos(\omega t_3 + \varphi_0) = U_m cos(\omega t_3) \cdot \cos\varphi_0 - U_m \sin(\omega t_3) \cdot \sin\varphi_0 \end{cases}$$

$$= \begin{cases} U_m cos(-\omega \Delta\mathrm{t}) \cdot cos\varphi_0 - U_m \sin(-\omega\Delta\mathrm{t}) \cdot \sin\varphi_0 \\ U_m cos(0) \cdot \cos\varphi_0 - U_m \sin(0) \cdot \sin\varphi_0 \\ U_m cos(\omega\Delta\mathrm{t}) \cdot cos\varphi_0 - U_m \sin(\omega\Delta\mathrm{t}) \cdot \sin\varphi_0 \end{cases} =$$

$$= \begin{cases} U_2 cos(\omega\Delta\mathrm{t}) + U_m \sin(\omega\Delta\mathrm{t}) \cdot \sin\varphi_0 \\ U_m cos\varphi_0 \\ U_2 cos(\omega\Delta t) - U_m \sin(\omega\Delta\mathrm{t}) \cdot \sin\varphi_0 \end{cases}.$$

Summing up the first and third equations of the system, we obtain

$$U_1 + U_3 = 2U_2 cos(\omega\Delta t) = 2U_m cos(\omega\Delta t) cos\varphi_0. \quad (1)$$

Where do we find the unknown frequency

$$\omega = \frac{1}{\Delta t}\left[\pm arccos\left(\frac{U_1 + U_3}{2U_2}\right) + 2\pi k\right], k = 0, \pm 1, \pm 2, \ldots \quad (2)$$

We show that only one value k = 0 is possible. Already at k = ± 1, the minimum value of the frequency modulus

$$\omega = 2\pi f = \frac{1}{\Delta t}\left|-arccos\left(\frac{U_1+U_3}{2U_2}\right) + 2\pi\right| = \frac{1}{\Delta t}\left|+arccos\left(\frac{U_1+U_3}{2U_2}\right) - 2\pi\right| \geq \frac{1}{\Delta t}[-\pi + 2\pi] = \frac{\pi}{\Delta t},$$

from here $\Delta t \geq \frac{1}{2f}$, which contradicts the requirements of the Kotelnikov theorem [4].

It is also easy to show that the inequality $0 \leq \frac{U_1+U_3}{2U_2} \leq 1$ is always satisfied. For physical reasons, you need to take a positive value of the frequency. Finally

$$\omega = \frac{1}{\Delta t}\left[arccos\left(\frac{U_1 + U_3}{2U_2}\right)\right]. \quad (3)$$

Subtracting the third from the first equation, we get

$$U_1 - U_3 = 2U_m sin(\omega\Delta t) sin\varphi_0.$$

In view of equation (1), we obtain

$$\frac{U_1 - U_3}{U_1 + U_3} = tg\varphi_0 \cdot tg(\omega\Delta t),$$

where do we find the solution for the initial phase of the oscillations at time t_2:

$$\varphi_0 = arctg\left[\frac{U_1 - U_3}{U_1 + U_3} \cdot ctg(\omega\Delta t)\right] + \pi k, k = 0, \pm 1, \pm 2, \dots \qquad (4)$$

The principle of determining k and eliminating ambiguity is shown in Fig. 1

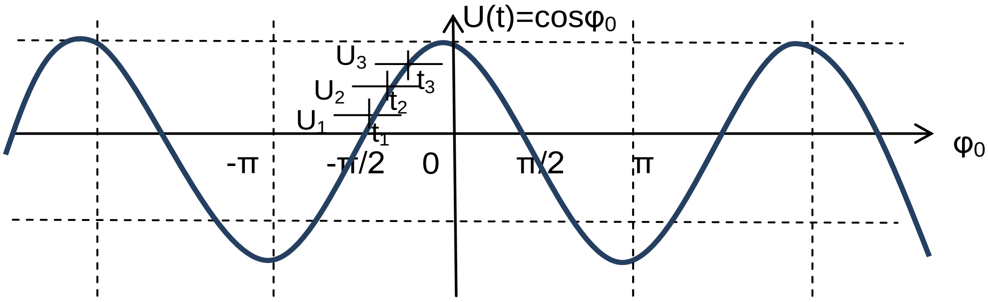

Figure 1. To the explanation of the choice of the value of k from the samples of signal for different initial phase φ_0

It is known that the main value of the arc tangent satisfies the inequality

$$-\frac{\pi}{2} < Arctg\left[\frac{U_1 - U_3}{U_1 + U_3} \cdot ctg(\omega\Delta t)\right] < \frac{\pi}{2}.$$

Therefore, for $-\frac{\pi}{2} < \varphi_0 < \frac{\pi}{2}$, when $cos\varphi_0 > 0, U_2 > 0$, and should be $k = 0$. In this case, the equality $ctg(arccosx) = \frac{x}{\sqrt{1-x^2}}$ is correct, using which, we get

$$\varphi_0 = arctg\left[\frac{U_1 - U_3}{U_1 + U_3} \cdot ctg(\omega\Delta t)\right] = arctg\left\{\frac{U_1 - U_3}{U_1 + U_3} \cdot ctg\left[arccos\left(\frac{U_1 + U_3}{2U_2}\right)\right]\right\} =$$

$$= arctg\left[\frac{U_1 - U_3}{\sqrt{4U_2^2 - (U_1 + U_3)^2}}\right]$$

As can be seen in Fig.1, for $-\pi < \varphi_0 < -\frac{\pi}{2}$, when $cos\varphi_0 < 0$, $U_2 < 0$, $U_1 < U_3$, and should be $k = -1$:

$$\varphi_0 = arctg\left[\frac{U_1 - U_3}{U_1 + U_3} \cdot ctg(\omega\Delta t)\right] - \pi.$$

For $\frac{\pi}{2} < \varphi_0 < \pi$, when $cos\varphi_0 < 0$, $U_2 < 0$, $U_1 > U_3$, and should be $k = +1$:

$$\varphi_0 = arctg\left[\frac{U_1 - U_3}{U_1 + U_3} \cdot ctg(\omega\Delta t)\right] + \pi.$$

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When programming the microcontroller, the marks for calculating φ_0 are the sign U_2 and the relationships between U_1 and U_3, which are visible in Fig.1.

Further, because $U_2 = U_m cos\varphi_0$, we can find an expression for calculating the signal amplitude

$$U_m = \frac{U_2}{cos\varphi_0} \tag{5}$$

where φ_0 calculated by the formula (4).

For $-\frac{\pi}{2} < \varphi_0 < \frac{\pi}{2}$ we use the expression $cos(arctgx) = \frac{1}{\sqrt{1-x^2}}$:

$$U_m = \frac{U_2}{cos\left\{arctg\left[\frac{U_1-U_3}{\sqrt{4U_2^2-(U_1+U_3)^2}}\right]\right\}} = \frac{2U_2\sqrt{U_2^2 - U_1U_3}}{\sqrt{4U_2^2 - (U_1 + U_3)^2}} \tag{6}$$

Thus, if the requirements of Kotelnikov's theorem are fulfilled, were obtained expressions (2...6), with the help of which it is possible to calculate the unknown amplitude, phase and frequency of a monochromatic harmonic signal from three ADC samples.

Estimation of accuracy of signal parameters recovery

It is known that the lower bound for the error variance of an unknown parameter in the sample is given by the Kramer – Rao inequality, or, in the multidimensional case, the Fisher information matrix [7 - 8]. The least error variance has the parameter estimate by the position of the maximum likelihood function (this is the best linear unbiased estimate). To obtain an estimate of error variance, it is necessary to construct a multidimensional likelihood function. Refinement estimates for error variance are also known using the Bhattacharia formulas [8].

In this case, the estimation of the signal parameters is obtained not from the analysis of the likelihood function, but by solving the corresponding equations, the roots of which are unknown parameters: frequency, phase and amplitude of the signal, formulas (2 ... 4).

For an important special case, if the passband of the preceding stages is several times greater than the frequency of the measured signal, then the spectral density of the noise power will have a corresponding wide band. Since, in accordance with the Khinchin – Kolmogorov theorem, the autocorrelation function of a signal is the Fourier transform of its power spectral density, the autocorrelation function of noise will rapidly decrease over a time several times smaller than the oscillation period of the signal under study. In addition, the quantization noise of different ADC samples is uncorrelated. Thus, noise voltages at times t_1, t_2, t_3 can be approximately considered uncorrelated.

Then, due to the action of additive uncorrelated Gaussian noises, the normal distribution law takes place [9]:

$$f(U_1, U_2, U_3) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma_1\sigma_2\sigma_3} e^{-\frac{(U_1-m_1)^2}{2\sigma_1^2}} \cdot e^{-\frac{(U_2-m_2)^2}{2\sigma_2^2}} \cdot e^{-\frac{(U_3-m_3)^2}{2\sigma_3^2}}$$

For example, the average of distribution of the angular velocity ω estimation, taking into account formula (3), is determined by the expression

$$M[\omega(U_1, U_2, U_3)] = \iiint_{-\infty}^{\infty} \left[\frac{1}{\Delta t}\arccos\left(\frac{U_1 + U_3}{2U_2}\right)\right] f(U_1, U_2, U_3) dU_1\, dU_2 dU_3 = M_\omega \ . \quad (7)$$

The error variance is calculated by the formula

$$D[\omega(U_1, U_2, U_3)] = \iiint_{-\infty}^{\infty} \left[\frac{1}{\Delta t}\arccos\left(\frac{U_1 + U_3}{2U_2}\right) - M_\omega\right]^2 f(U_1, U_2, U_3) dU_1\, dU_2 dU_3 = D_\omega \, . \quad (8)$$

Since the exact calculation using formulas (7, 8) is quite complicated, an approximate method was used to calculate the average of distribution and error variance of the parameter estimate.

1. The frequency estimation error

We expand the objective function (3) $\omega(U_1, U_2, U_3)$ in a Taylor series in the vicinity of the point of the true voltage values (U_{10}, U_{20}, U_{30}) according to the powers of error of the voltage measurement and keep the first four terms of series:

$$\omega(U_1, U_2, U_3) = \omega(U_{10} + \Delta U_1, U_{20} + \Delta U_2, U_{30} + \Delta U_3) \approx$$
$$\approx \omega(U_{10}, U_{20}, U_{30}) + \Delta U_1 \frac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_1} + \Delta U_2 \frac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_2} +$$
$$+\Delta U_3 \frac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_3} \, . \quad (9)$$

Hence, the average of distribution of the measuring error of frequency *ω*

$$M_\omega = M[\omega(U_{10} + \Delta U_1, U_{20} + \Delta U_2, U_{30} + \Delta U_3)] - \omega(U_{10}, U_{20}, U_{30}) \approx$$
$$\approx M(\Delta U_1) \frac{\partial \omega(U_{10},U_{20},U_{30})}{\partial U_1} + M(\Delta U_2) \frac{\partial \omega(U_{10},U_{20},U_{30})}{\partial U_2} + M(\Delta U_3) \frac{\partial \omega(U_{10},U_{20},U_{30})}{\partial U_3} \quad (10)$$

If random thermal noise has a zero constant component, then $M(\Delta U_1) = M(\Delta U_2) = M(\Delta U_3) = 0$. Then from the above formula we get that if we keep the first four terms of series, then the frequency estimate is unbiased, $M_\omega = 0$.

Similarly, we find the error variance of the frequency for uncorrelated samples of the ADC samples:

$$D_\omega = D[\omega(U_{10} + \Delta U_1, U_{20} + \Delta U_2, U_{30} + \Delta U_3)] \approx$$
$$\approx D\left[\begin{array}{c}\omega(U_{10}, U_{20}, U_{30}) + \Delta U_1 \dfrac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_1} + \Delta U_2 \dfrac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_2} + \\ +\Delta U_3 \dfrac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_3}\end{array}\right] =$$
$$= D(\Delta U_1)\left[\frac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_1}\right]^2 + D(\Delta U_2)\left[\frac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_2}\right]^2 +$$
$$+D(\Delta U_3)\left[\frac{\partial \omega(U_{10}, U_{20}, U_{30})}{\partial U_3}\right]^2 . \quad (11)$$

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Denote

$$4U_2^2-(U_1+U_3)^2=A,\quad U_2^2-U_1U_3=B.$$

For function (3), after obvious but cumbersome transformations, we obtain:

$$\frac{\partial\omega(\mathrm{U}_{10},\mathrm{U}_{20},\mathrm{U}_{30})}{\partial\mathrm{U}_1}=-\frac{1}{\Delta t\sqrt{A}}$$

$$\frac{\partial\omega(U_{10},U_{20},U_{30})}{\partial U_2}=\frac{U_1+U_3}{\Delta tU_2\sqrt{A}}$$

$$\frac{\partial\omega(U_{10},U_{20},U_{30})}{\partial U_3}=-\frac{1}{\Delta t\sqrt{A}}$$

2. The signal phase estimation error

Similarly to what was done in Sec. 1 for estimating the error in measuring the frequency, we expand the objective function (4) $\varphi_0(U_1,U_2,U_3)$ in a Taylor series in the vicinity of the point of the true voltage values (U_{10},U_{20},U_{30}) according to the powers of measurement error voltages. Keep the first four terms of series, we obtain expressions for estimating the phase measurement error. These expressions are similar to formulas (10) and (11).

With a linear restriction, the estimate of the average of distribution of the error in the phase measurement is also unbiased, and the expression for the variance has the form

$$D_{\varphi_0}=D[\varphi_0(U_{10}+\Delta U_1,U_{20}+\Delta U_2,U_{30}+\Delta U_3)]\approx$$
$$=D(\Delta U_1)\left[\frac{\partial\varphi_0(U_{10},U_{20},U_{30})}{\partial U_1}\right]^2+D(\Delta U_2)\left[\frac{\partial\varphi_0(U_{10},U_{20},U_{30})}{\partial U_2}\right]^2+$$
$$+D(\Delta U_3)\left[\frac{\partial\varphi_0(U_{10},U_{20},U_{30})}{\partial U_3}\right]^2. \qquad (12)$$

For function $\varphi_0(U_1,U_2,U_3)$ (4) after transformations we get:

$$\frac{\partial\varphi_0(\mathrm{U}_{10},\mathrm{U}_{20},\mathrm{U}_{30})}{\partial\mathrm{U}_1}=\frac{A+(U_1^2-U_3^2)}{4\sqrt{AB}}$$

$$\frac{\partial\varphi_0(U_{10},U_{20},U_{30})}{\partial U_2}=-\frac{4U_2(U_1-U_3)}{\mathrm{A}\sqrt{A}}$$

$$\frac{\partial\varphi_0(U_{10},U_{20},U_{30})}{\partial U_3}=\frac{(U_1^2-U_3^2)-A}{4\sqrt{AB}}$$

3. The signal amplitude estimation error

A similar expansion of the objective function (6) $U_m(U_1,U_2,U_3)$ in a Taylor series in the vicinity of the point of the true voltage values (U_{10},U_{20},U_{30}) according to the powers of voltage measurement errors and the calculation of the variance of the amplitude estimate gives the result:

$$D_{U_m} = D[U_m(U_{10} + \Delta U_1, U_{20} + \Delta U_2, U_{30} + \Delta U_3)] \approx$$

$$= D(\Delta U_1)\left[\frac{\partial U_m(U_{10}, U_{20}, U_{30})}{\partial U_1}\right]^2 + D(\Delta U_2)\left[\frac{\partial U_m(U_{10}, U_{20}, U_{30})}{\partial U_2}\right]^2 +$$

$$+D(\Delta U_3)\left[\frac{\partial U_m(U_{10}, U_{20}, U_{30})}{\partial U_3}\right]^2. \quad (13)$$

For function $U_m(U_1, U_2, U_3)$ (6) after transformations we get:

$$\frac{\partial U_m(U_{10}, U_{20}, U_{30})}{\partial U_1} = -\frac{U_2 U_3}{\sqrt{A}\sqrt{B}} + \frac{2U_2(U_1 + U_3)\sqrt{B}}{A\sqrt{A}};$$

$$\frac{\partial U_m(U_{10}, U_{20}, U_{30})}{\partial U_2} = -\frac{8U_2^2\sqrt{B}}{A\sqrt{A}} + \frac{2U_2^2 + 2B}{\sqrt{AB}};$$

$$\frac{\partial U_m(U_{10}, U_{20}, U_{30})}{\partial U_3} = -\frac{U_1 U_2}{\sqrt{A}\sqrt{B}} + \frac{2U_2(U_1 + U_3)\sqrt{B}}{A\sqrt{A}}.$$

If the root-mean-square (RMS) values of noise voltages in the ADC samples $\sigma_u = \sqrt{D(\Delta U_i)}$ are known, then the RMS values of the error in determining the signal amplitude can be calculated using the above formulas.

It is important to note that for sufficiently large signal-to-noise ratios, until the stability of the solutions of equation (4) is violated, the proposed estimation technique yields results that closely coincide with the Kramer-Rao estimates.

4. Numerical calculation of errors in determining the noisy sinusoidal signal parameters

The dependence of the error in calculating the unknown signal parameter from the ratio of the quantization interval *Δt* and the oscillation period *T* of the digitized signal was analyzed.

The calculation was made for the harmonic signal $(t) = U_m cos(\omega t + \varphi_0)$ at a frequency $f = 5139{,}6$ Hz, the RMS noise value $\sigma_n = 1$ mV, the sampling interval Δt = 30 μs.

The calculation results for different values of the phase φ_0 are shown in table 1.

Table 1

RMS errors of measuring the frequency, phase and amplitude of harmonic signal

φ_0, deg	-100	-80	-60	-40	-20	0	20	40	60	80	100
σ_f, Hz	33,71	33,67	11,70	7,64	6,23	5,85	6,23	7,64	11,70	33,67	33,71
σ_φ, deg	2,759	2,752	0,295	0,102	0,058	0,049	0,058	0,102	0,295	2,752	2,759
σ_{Um}, mV	6,421	5,963	1,483	0,830	0,939	1,016	0,961	1,020	1,875	6,412	5,972

Table 2 shows the calculation results for different ADC sampling frequencies F_d for such data: the initial phase of the average readout $\varphi_0 = \frac{\pi}{4}$, the frequency *f* = 5139.6 Hz, and the RMS noise value $\sigma_n = 1$ mV.

Table 2

The influence of the sampling frequency on the RMS errors of single measurement of the frequency, phase and amplitude of the harmonic signal

Parameter	The ratio of the interval between samples to the period of the signal oscillation, Δt/T									
	0,025	**0,075**	**0,125**	**0,175**	**0,225**	**0,275**	**0,325**	**0,375**	**0,425**	**0,475**
F_d, kSPS	205,58	68,528	41,117	29,369	22,843	18,689	15,814	13,706	12,093	10,820
σ_f, Hz	360,45	38,77	13,13	6,25	3,78	3,09	3,37	4,38	6,84	18,97
σ_φ, deg	3,511	0,417	0,172	0,108	0,088	0,088	0,108	0,172	0,417	3,511
σ_{Um}, mV	35,31	3,685	1,414	1,038	0,926	0,787	0,577	1,000	4,599	51,83

The ratio of the error in the calculation of the amplitude to the RMS noise voltage $\frac{\sigma_{Um}}{\sigma_n}$ for various initial phases of the second sample φ_0 (the phase can be random in the interval 0 ... 2π) is shown in Fig. 2 ... 4.

Obviously, as $\frac{\Delta t}{T} \to 0{,}5$, the error tends to infinity (the conditions of the Kotelnikov theorem are not satisfied). If the sampling interval is small, $\frac{\Delta t}{T} \to 0$, then the values of the voltage samples U_1, U_2, U_3 differ little from each other and the error also increases.

It can be concluded that at $\frac{\Delta t}{T} \approx 0{,}175 \ldots 0{,}375$ the error in calculating the amplitude $\sigma_{Um} \approx (0{,}4 \ldots 1{,}5)\sigma_n$, there is no noticeable deterioration in the accuracy of determining the amplitude due to the described algorithm. But the number of calculated values of amplitudes decreases three times in comparison with the rate of digitization.

By complicating the algorithm for selecting triples of samples U_1, U_2, U_3 (it is advisable to choose not three consecutive samples, but spaced apart from each other by the interval $\frac{\Delta t}{T} \approx 0{,}20 \ldots 0{,}35$ samples from the array of more frequent measurements), it will be possible to increase the number of measurements for one period of oscillation of the resonator.

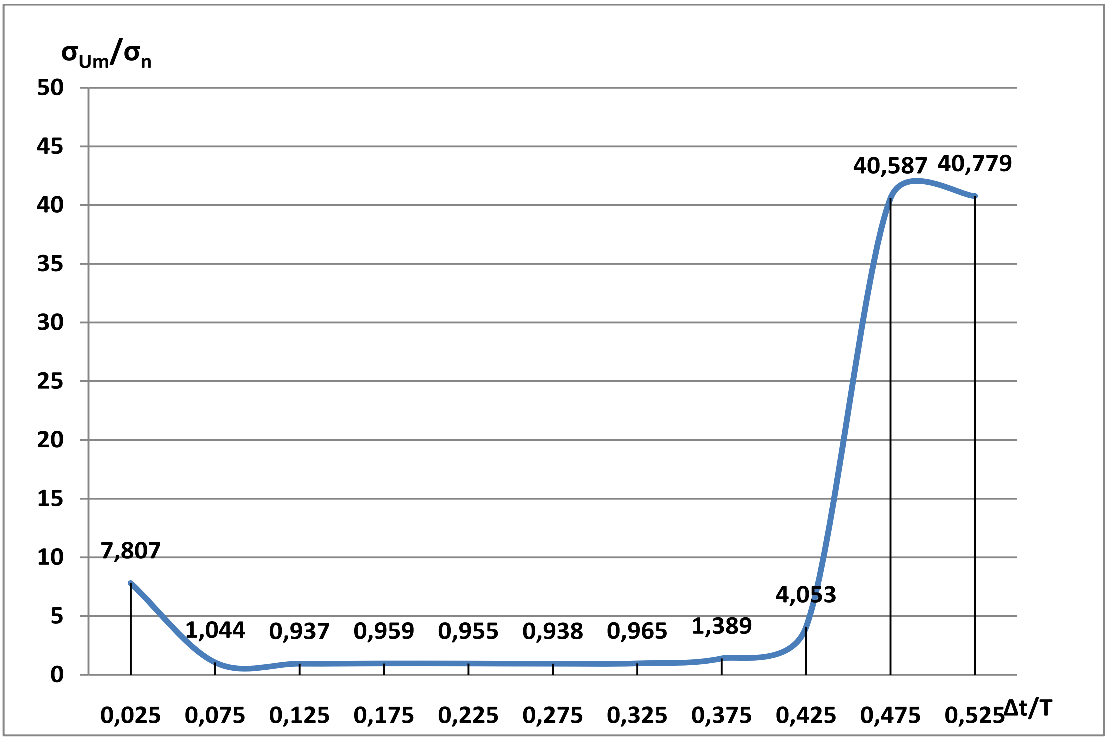

Figure 2. The dependence of the error in calculating the signal amplitude from the ratio of the sampling interval and the oscillation period $\frac{\Delta t}{T}$ for $\varphi_0 = \pi/8$

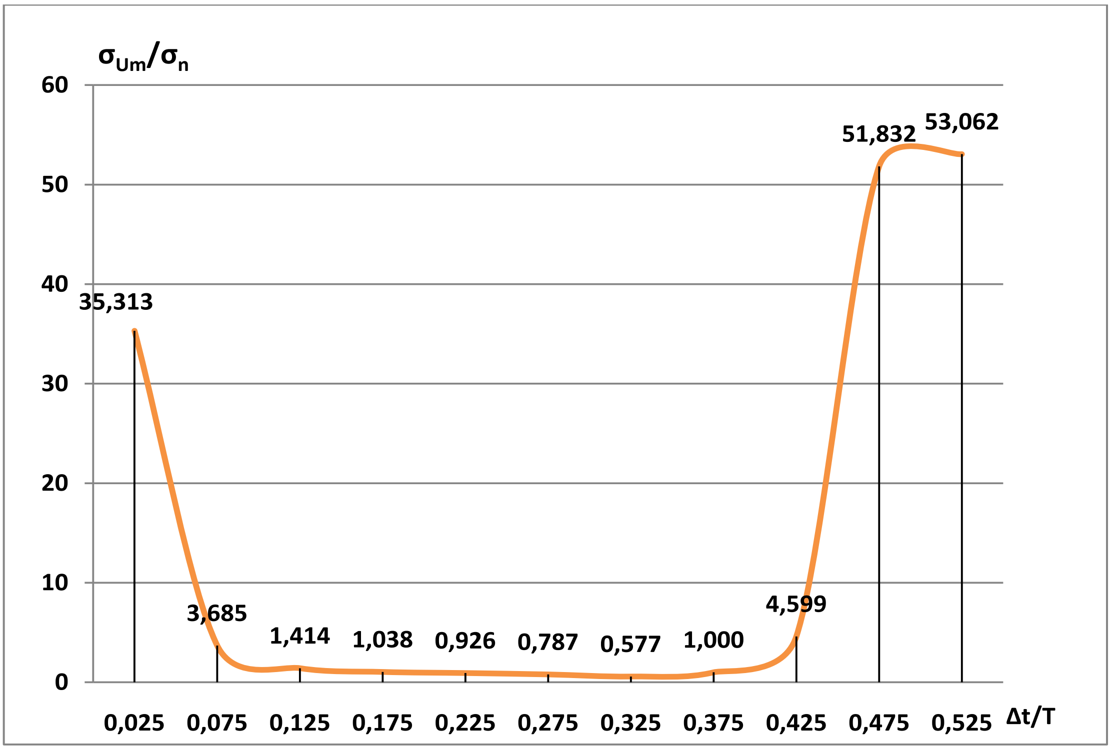

Figure 3. The dependence of the error in calculating the signal amplitude on the ratio of the sampling interval and the oscillation period $\frac{\Delta t}{T}$ for $\varphi_0 = \pi/4$

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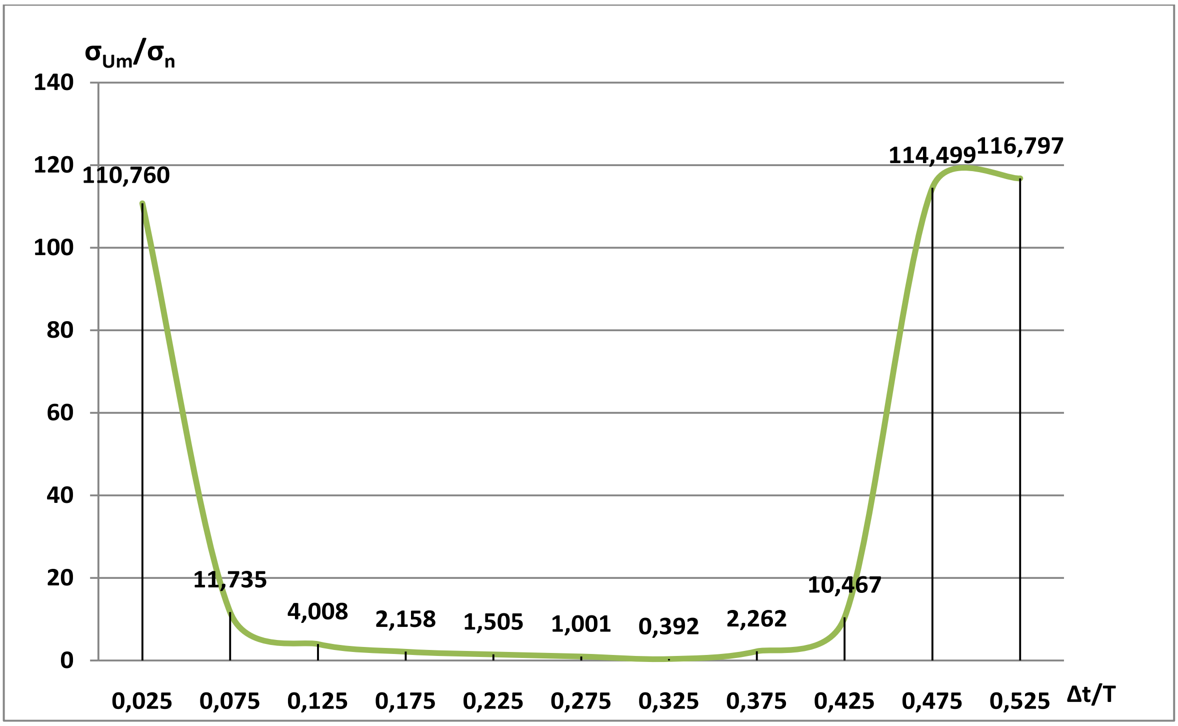

Figure 4. The dependence of the error in calculating the signal amplitude on the ratio of the sampling interval and the oscillation period $\frac{\Delta t}{T}$ for $\varphi_0 = 3\pi/8$

Conclusion

1. When $\varphi_0 \to \pm\pi/2$, the measurement errors of all parameters are increase significantly. It is advisable to take this into account during secondary processing the results of measurements series. It is advisable to calculate the corresponding weights of individual measurements, depending of the φ_0 estimate.

2. A potent dependence of the error in measuring the frequency, phase, and amplitude of the signal from the ratio of the sampling period to the period of the estimated signal is observed. From the point of view of a minimum of errors, the optimal values of Δt/T are in the range of 0.175 ... 0.375.

3. The error in determining the amplitude in single measurement is commensurate with the RMS noise for the phase range $-\pi/3 < \varphi_0 < \pi/3$.

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