

## MODELING AND OPTIMIZATION OF A MULTIPHASE SEQUENTIAL TECHNOLOGICAL PROCESS

*Abstract.* This paper solves two problems of optimizing the operation of a multiphase sequential process (STP): innovative equipment upgrades to increase productivity and determining the optimal number of similar equipment at each phase of the production process.

*Keywords:* technological process, sequential cycle, Queuing system, simulation model, optimization, calculated relations.

### Introduction

In the conditions of competition and instability, production companies forced continuously improve the production process and develop new innovative technologies and equipment in order to maintain their positions in the market. An important role in this play by improving the methods of planning and managing the process of production development, development of innovative products and forecasting measures to adapt to changing requirements of the external and internal market. Technology management aimed at solving the problem of selecting technological processes and ensuring a certain technological potential. The main content of the production development strategy is the following – innovation of an integrated balance between the equipment, the workforce and the manufactured products [1–3,6].

The introduction of modern innovative technologies and the deployment of new production is a set of measures for the acquisition, deployment, development and organization of operation of complex equipment. In conditions of fierce competition, this process should satisfy a number of indicators, which is impossible without the use of mathematical methods of optimization and application of software [8].

To date, the following main types of innovations can be identified [4]:

- the product innovation;
- the innovation of technological processes (TP);
- the organizational innovation.

The main component of reliability, competitiveness, product cost, etc. of any production is the technological process. Hence, the main characteristics of the product that provide its market advantages depend on the level and perfection of its manufacturing technology. Innovation of technological processes makes it possible to ensure the growth of the enterprise's production potential, increase labor productivity and save resources.

A set of consistently linked unified models of technological operations allows you to build process models of individual technological processes and evaluate such characteristics as inter-shop and intra-shop loading of equipment, performance, rhythm, equipment, etc.

Changing certain parameters of a technological process, it possible achieve the optimal ratio of production cycle time and product costs. In fact, solve the task of finding the best solution for various options of equipment and tooling *TP*, the timing and costs of implementation of innovative development plans of production, which is largely determined by the level of flexibility of the production process [5,6].

Next, it necessary to analyze the parameters that determine the effectiveness of its operation within this production system for the selected version of the *TP* structure. To solve this problem, it is necessary to evaluate the dynamics of the *TP* operation based on the selected version of the *TP* structure model.

### Modeling and evaluation of sequential process characteristics

To evaluate the characteristics of a *sequential technological process (STP)*, it proposed to use the mathematical apparatus of *Queuing systems (QS)*, which allows you to simulate its operation, taking into account the availability of time and material resources necessary to perform production tasks [5,6]. At the same time, you can estimate the actual execution time of not only one *TP*, but also their sequence.

Let consider a production system for manufacturing products produced in small and medium-sized batches. A modern *complex production system (CPS)* includes groups of machines or production modules with automatic processing of parts, an automated system for loading and unloading workpieces, a conveyor system for transporting workpieces, etc.

In other words, *CPS* is a complex system of interrelated elements and modules of the production process, technological processes that form a single whole and perform functions for the production of industrial products. She has linear ones and the branched structure of the *STP* implementation, which takes into account the parallel use of several units of the same type of equipment. The linear structure of the *STP* is typical for production lines (automatic and non-automatic) of the conveyor type [2,3]. In General, production with a sequential multiphase process chain can graphically represented as a sequential chain of multichannel *QS* with a queue in each production phase (Fig.1).

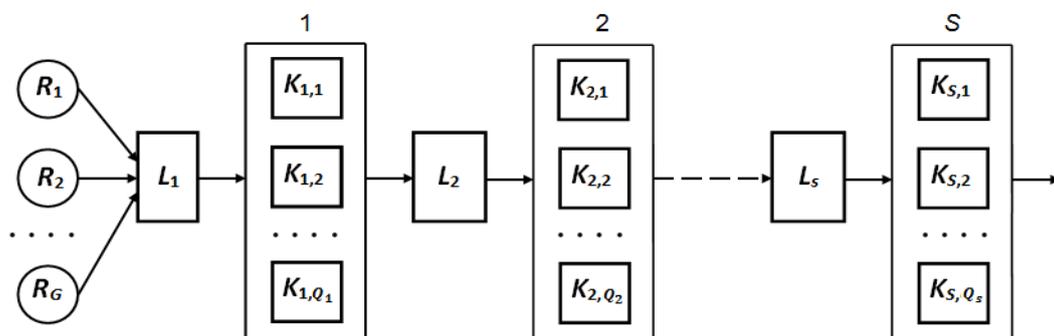


Figure 1. Sequential process flow scheme

Here  $R_1 \dots R_G$  – the sources of service requests;  $K_{1,G} \dots K_{S,G}$  – the service channels;  $S$  – number of service phases;  $Q_1 \dots Q_S$  – the number of channels in each service phase;  $L_1 \dots L_S$  – the service request queues.

### Statement of the problem simulation

The multi-phases structure of *STP* considered (Fig.1) as *QS* with a Poisson input flow (receipt of parts), a fixed service time, and parallel functioning service nodes (machines or production modules) [1,2,5].

The output stream is the flow of served requests (manufactured items). Queue discipline not regulated. In addition, no matter how many requirements (parts) are received at the input of the service system (machines or production modules), this system (*STP*) cannot process more than  $N$  requirements (parts) at once.

Requests (parts) that are not included in the intermediate technological "capacity" for waiting forced to be serviced via a different channel (machine or production module). To evaluate the characteristics of the *STP* obtained by repeatedly modeling the above-described *QS* model, it is necessary to determine the criteria for the effectiveness of the *STP* operation and its cost analysis models.

Thus, each phase of the *STP* considered as a multi-channel *QS*, which receives a Poisson flow of requests with an intensity of  $\lambda$ , and the service intensity of each channel is  $\mu$ , the maximum possible number of places in the queue is limited by the value  $m$ . The discrete states of the *QS* determined by the number of requests received in the system, which can written as:

- all channels are free;
- only one channel is busy (any channel);
- only two channels occupied (any);
- all  $n$  channels are busy.

As long as the *QS* is in any of these states, there is no queue. After all service channels are busy, subsequent requests form a queue, thereby determining the future state of the system [7–9]:

- all channels are busy and one request is in the queue;
- all channels are busy and two applications are in the queue;
- all  $n$  channels and all  $m$  places in the queue occupied.

Graph of States of an  $n$ -channel *QMS* with a queue bounded by  $m$  places in Fig.2.

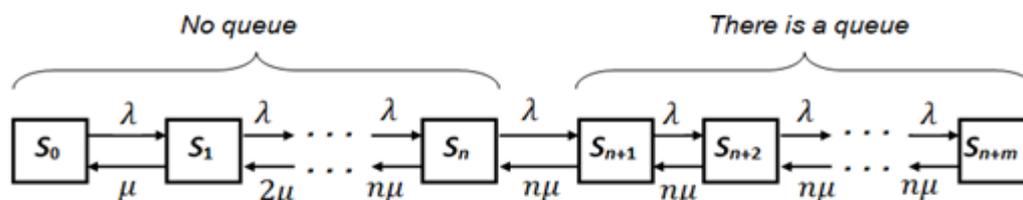


Figure 2. Graph of States of an  $n$ -channel *QMS* with a queue bounded by  $m$  places

The transition of the *QS* to the state with a large number determined by the flow of incoming requests with intensity  $\lambda$ , whereas, by condition,  $m$  identical channels with a service flow intensity  $\mu$  equal for each channel take part in servicing these requests. We write the expressions for the limiting probabilities of states[9]:

$$\begin{aligned} p_1 &= \frac{\rho}{1!} p_0, \\ p_2 &= \frac{\rho^2}{2!} p_0, \\ &\dots \\ p_k &= \frac{\rho^k}{k!} p_0, \quad 0 \leq k \leq n, \\ &\dots \\ p_n &= \frac{\rho^n}{n!} p_0. \end{aligned} \quad (1)$$

$$p_0 = \left[ 1 + \frac{\rho}{1!} + \frac{\rho^2}{2!} + \dots + \frac{\rho^n}{n!} + \frac{\rho^n \rho - \left(\frac{\rho}{n}\right)^{m+1}}{n! \left(1 - \frac{\rho}{n}\right)} \right]^{-1} \quad k = 0;$$

$$p_0 = \left[ \sum_{k=0}^n \frac{\rho^k}{k!} + \frac{\rho^n}{n!} \frac{\rho - \left(\frac{\rho}{n}\right)^{m+1}}{1 - \frac{\rho}{n}} \right]^{-1} = \left[ \sum_{k=0}^n \frac{\rho^k}{k!} + \frac{\rho^{n+1}}{n!(n-\rho)} \left(1 - \left(\frac{\rho}{n}\right)^m\right) \right]^{-1}, \quad (2)$$

where  $\rho = \frac{\lambda}{\mu}$ .

Average number of requests in the queue:

$$L_q = \frac{\rho^{m+1} p_0 \left[ 1 - (m+1 - m \frac{\rho}{n}) \left(\frac{\rho}{n}\right)^n \right]}{n n! \left(1 - \frac{\rho}{n}\right)^2}. \quad (3)$$

The average number of requests for service is equal to the average number of busy channels:

$$L_s = K = \rho \left( 1 - \frac{\rho^{n+m}}{n^m n!} p_0 \right). \quad (4)$$

Under modeling of branched structure of the *STP* for each phase is calculated:

- the average waiting time for part processing;
- the average equipment downtime;
- the maximum queue length (volume of intermediate storage of items);
- the machine utilization;
- the average time of technological operation;
- the maximum processing time.

Also under modeling, the overall *STP* indicators evaluate:

- the total production time of the planned volume;
- the time of completion of *STP*;
- the total equipment utilization rate by time:

- the total utilization rate of the equipment depending on the volume of production.

### Criteria for the effectiveness of a sequential *TP*

The development of performance criteria is the ultimate goal of analyzing the functioning of technological processes, including *STP*. Under the stationary conditions considered for each phase of the production process the following operational characteristics of *QS* for evaluating *TP*[1,3]:

- $p_n$  – the probability that there are  $n$  service requests (parts) in the production process, formula (1);
- $L_S$  – average number of requests (parts) in the production process, formula (4);
- $L_q$  – average number of requests (parts) in the service queue, formula (3);
- $W_S$  – average length of stay of the application (parts) in the production process;
- $W_q$  – average length of stay request (parts) in the service queue in the production process.

It's obvious that

$$L_S = \sum_{n=0}^{\infty} n p_n, \quad L_q = \sum_{n=0}^{\infty} (n - c) p_n. \quad (5)$$

Here  $n$  – the total number of parts in the *STP*;  $c$  – number of machine tools or production modules.

In particular, if the frequency of receipt requests (parts) in the production process is equal to  $\lambda$ , then we have

$$L_S = \lambda W_S; \quad L_q = \lambda W_q. \quad (6)$$

The above relations are also true under much less rigid assumptions that do not impose any special restrictions on either the distribution of moments of successive receipts of parts or the distribution of the duration of their.

When the frequency of receipt of requests (parts) is equal to  $\lambda$ , but not all requests (parts) can served in the *STP* due to the restriction on the volume of the intermediate warehouse, the ratio (1) modified. To do this, define a new value for the parameter  $\lambda$ , which would allow us to take into account only those requirements that are really "allowed" in the *STP*.

Then, introducing  $\lambda_{eff}$  is the effective frequency of receipts. That is, the number of requirements that are actually allowed in the waiting block of the service system per unit of time, we will

$$\lambda_{eff} = \beta \lambda; \quad 0 < \beta < 1. \quad (7)$$

This means that only a part of the incoming requests (details) actually received for service in the *STP*.

If the average service speed is equal to  $\mu$  and, therefore, the average service duration is equal to  $1/\mu$ , then the following ratio is true:

$$W_S = W_q + 1/\mu. \quad (8)$$

Multiplying the left and right parts of this ratio by  $\lambda$ , we will

$$L_S = L_q + 1/\mu. \quad (9)$$

The latter relation remains valid even if  $\lambda$  replaced by  $\lambda_{eff}$ . In this case, for  $\lambda_{eff}$  can write

$$\lambda_{eff} = \mu (L_S - L_q). \quad (10)$$

Knowing  $p_n$ , you can determine the value of all the main operational characteristics. In particular,  $p_n$  for multichannel QS with the queue  $p_n$  and other characteristics determined according to (1) – (4).

For the smooth operation of multiphase *PTP* (Fig.1), it is necessary that the following condition is met for each phase:  $\lambda_{s+1} \leq \mu_s$ . In addition, if the conditions regarding the flows of receipt and service of applications in the real *STP* not met, multiple simulations performed to determine the  $p_n$  and other characteristics [2,5,9].

### **Cost models for sequential TP analysis**

Cost models of *QS* aimed at determining the level of functioning of the *STP*, which achieves a "compromise" between the following two economic indicators:

- profit from the implementation of the planned production volume;
- loss of profit due to delays in the production process and restrictions on the volume of production, which are determined by the corresponding production capacity (equipment, personnel).

Obviously that the increase in the functional capacity of the *STP* should lead to a reduction in the time spent in the technological "capacity", that is, leads to an increase in production volumes.

This means that as the production costs associated with the use of equipment increase due to an increase in the technological level of production (modernization of equipment), the losses associated with downtime, expressed in economic terms, will decrease.

Hence, two tasks of organizing the production process become relevant: the task of determining the optimal composition of *PTP* equipment and the task of determining the optimal number of the same type of *STP* equipment. Let consider the solution of these problems using the results of simulation.

### **Cost models for sequential TP analysis**

The solution to this problem is a very important assessment to ensure an appropriate details servicing speed characterizing *STP* performance. In fact, the solution to this problem related to the search for a compromise between more expensive and more productive equipment and the costs of its operation [10,11].

Assumed that the speed of service is subject to regulation, that is, it is possible to update equipment at each stage of the production process. Necessary to determine the optimal value of the service speed based on the constructed cost model.

Consider a situation where no more than  $N$  details can be in the technological "capacity" of service expectations. In this problem, the value of  $N$  considered as a control variable, the optimal value of which (together with  $\mu$ ) is determined by minimizing the total cost of modernization:

$$K_M(\mu, N) = k_1 \mu + k_2 L_s + k_3 N + k_4 \lambda p_n, \quad (11)$$

где  $k_1$ – service costs assigned to a time unit;  $k_2$ – the given costs per unit of time, due to the presence of details for maintenance in standby mode;  $k_3$ – calculated per unit of time cost of increasing the capacity of the technological "capacity" of waiting for parts for service;  $k_4$ – economic losses associated with the inability to include in the technological "capacity" of waiting for parts to service another detail for subsequent processing;  $\lambda p_n$  – the number of parts "lost" per unit of time.

Since  $\mu$  is a continuous quantity, its optimal value can be obtained by equating to zero in expression (9) the first derivative of  $K_M(\mu)$  by  $\mu$  for a given number  $N$  and vice versa.

### Determining the optimal number of the same type of equipment

The result of solving this problem is to obtain a compromise solution between increasing the number of equipment and increasing the cost of their maintenance (operation, maintenance and depreciation). At the same time, savings can be achieved by reducing the downtime of other equipment in the *PTP*, and therefore, it is possible to increase the volume of production.

The cost model of multichannel *QS* in this case should be focused on determining the optimal number of servicing machines or production modules  $c$ . Assume that the values of  $\lambda$  and  $\mu$  are fixed. The integral cost of indicators given by the formula

$$K_c(c) = k_1 c + k_2 L_s(c), \quad (12)$$

where  $L_s(c)$  – the average number of machined parts in the production system.

The optimal value of  $c_{opt}$  can be found from the condition

$$K_c(c - 1) > K_c(c) \text{ и } K_c(c + 1) > K_c(c), \quad (13)$$

what is equivalent to inequality

$$L_s(c) - L_s(c + 1) < C_1 / C_2 < L_s(c - 1) - L_s(c). \quad (14)$$

The value  $k_1/k_2$  is a pointer to where the search for the optimal value of  $c$  should begin.

The stages of a generalized algorithm for solving this problem using the above formulas are:

**Stages 1.** Determined the average number of parts in the queue for the original number of equipment.

**Stages 2.** Determined the loss of working time in terms of value.

**Stages 3.** Make an assumption about an increase in equipment by one unit.

**Stages 4.** Determined the waiting time in the queue when the number of equipment increases.

**Stages 5.** Comparison of the additional costs of using additional equipment with the saved time for the technological process.

**Stages 6.** If the increase in the number of equipment not have a positive effect in relation to the time to complete the technological process, leave the original number of equipment. Otherwise, increase the amount of equipment per unit and proceed to **Stage1**.

### Conclusion

Propose criteria for evaluating the characteristics of a multiphase sequential process with a branched structure using the mathematical apparatus of a multi-channel Queuing system with waiting and queue restriction. On this model, the criteria for evaluating the work of the *STP* formed and cost models synthesized for solving the problems of innovation and optimization of the *STP*. For each phase of the production process, the problem of evaluating the effectiveness of innovative equipment upgrades and the problem of selecting the optimal amount of equipment has solved. If there is a discrepancy between the flow of receipt and processing of parts of this mathematical model, the values of the initial criteria determined by the results of a certain number of repeated simulation of the *STP* operation.

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