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OPTIMAL STABILIZATION OF THE MOVEMENT OF AN UNMANNED AERIAL VEHICLE WITH PARAMETRIC UNCERTAINTY

Annotation: This article proposes a modal approach to the synthesis of robust systems for optimal stabilization of the program flight path of a UAV under conditions of uncertainty of its state parameters. The synthesis of a modal robust controller is implemented using the principle of guaranteed dynamics, which allows ensuring not only stability, but also the required quality of control.

Keywords: unmanned aerial vehicle, uncertainty of parameters, linear-quadratic optimization, the principle of guaranteed dynamics, modal synthesis

Introduction

The technical level of use of unmanned aerial vehicles (UAVs), achieved in recent years, allows them to be used for any activity. One of the most important tasks is the high-quality implementation of the entire flight program, regardless of the influence of random factors. Therefore, the task of analyzing and synthesizing optimal UAV motion stabilization systems is certainly relevant. Thus, one of the most important areas in UAV control is the synthesis of stabilization systems under conditions of uncertainty. This is due to the inability in some cases to accurately determine the parameters of the UAV model, the action of uncontrolled disturbances, etc. That is why there is a need to create robust systems for stabilizing the program movement of the UAVs, which would ensure the necessary quality of UAV operation in these conditions.

Overview of existing solutions

Many approaches have been proposed to solve this problem [1-8, 14]. For example, in [4, 5], the parameters of the control device are chosen in such a way as to ensure that the system is insensitive to arbitrary unknown perturbations. Some methods of dynamic compensation of bounded perturbations are considered in [5-8]. In these works, a signal is distinguished that carries information about external and parametric perturbations of the system in order to compensate for their influence on the regulated variable. In addition, the urgent task is to choose among a set of stabilizing regulators one that optimizes a certain criterion that characterizes the quality of control.

The most common solution is the $H\infty$ optimization method, which consists in constructing a stabilizing regulator for systems with perturbations. Regulators synthesized using this optimality criterion ensure the stability of a closed system and minimal sensitivity to disturbances. In [4], for the design of a suboptimal regulator, the control effect is divided into two components: optimal control, which minimizes the specified quality functionality, and a component that compensates for the uncertainties of the control system. The goal of management is to minimize the integral quality criterion, and compensation for uncertainties in the object is based on the approach proposed in [5]. In [11, 14], it is proposed to use the principle of guaranteed dynamics to ensure stability.

This article is a further development of these works. The article is devoted to the development of a robust modal approach using the principle of guaranteed dynamics to ensure reliable stability of UAV software movement in stabilization modes with specified quality indicators [1, 2]. As a result, the problem of synthesizing a robust linear-quadratic controller with guaranteed dynamics of transients in motion stabilization modes of UAV software, the dynamics of which is described by a linear stationary model, is solved. According to [1,2, 14], the synthesis of an optimal reliable regulator is carried out as follows.

Problem statement

Let a fully controlled and fully observed linear dynamic model of UAV motion with parameter uncertainty be described in stabilization modes by a system of Linear Differential Equations of the form:

$$\dot{\bar{x}}(t) = (A + \Lambda)\bar{x}(t) + B\bar{u}(t), \tag{1}$$

where $\bar{x}(t)$ – n-dimensional vector of the system state; $\bar{u}(t)$ – m − dimensional control vector; A, B – matrices of coefficients of the linear UAV model and control intensity, respectively, by dimension $(n\times n)$, $(n\times m)$; Λ – unknown real matrix function of dimension uncertainties (n×n).

It is necessary to determine the optimal control $\bar{u}(t)$, which translates system (1) from a given initial state $\bar{x}(t_0) = x_0$ to a final state $\bar{x}(\infty) = 0$ and minimizes the quadratic functional of the form:

$$I_\sigma = \int_{t_0}^{t_k} [\bar{x}^{\mathrm{T}}(t) Q \bar{x}(t) + \bar{u}^{\mathrm{T}}(t) R \bar{u}(t)] dt, \tag{2}$$

where $t_0 = 0$; $t_k = \infty$, a Q і R – positively determining the matrix by dimension, respectively, (n×n) and (m×m).

Solving the problem

In the above statement, the problem of stabilization of linear dynamical systems with uncertainty in parameters refers to linear-quadratic optimization problems, which is reduced to solving a nonlinear algebraic Riccati equation to determine unknown coefficients in the optimal control law. This law is a linear combination of state variables of the desired dynamical system. Depending on the type of uncertainty Matrix Λ, there are two main approaches to solving the stabilization problem associated with solving the Riccati equation [9,10].

In the first case, a system of the form is considered

$$\dot{\bar{x}}(t) = [A + \Lambda(\mu)]\bar{x}(t) + B\bar{u}(t), \tag{3}$$

where $\lim_{\mu\to 0} \Lambda(\mu) = 0$, a μ – parametric uncertainty satisfying inequalities

$$\|\Lambda(\mu)\| \leq I_A \|\mu\|. \tag{4}$$

Then, according to [9], the optimal control can be represented as:

$$\bar{u}(t) = -(K + k)\bar{x}(t), \tag{5}$$

where K – matrix of coefficients of the optimal stabilization law of the system (5) in the absence of an uncertainty matrix Λ; k – matrix of coefficients of compensation for the influence of uncertainties on the parameters of the system (1), defined as

$$k = \lim_{\varepsilon\to 0} \varepsilon^{-1} B^{\mathrm{T}} P(\varepsilon) = 0, \quad P(\varepsilon) \geq 0. \tag{6}$$

In Formula (6) $P(\varepsilon)$ it is determined from the Lurie-Riccati equation [9]

$$A^{\mathrm{T}} P(\varepsilon) + P(\varepsilon)\mathrm{A} - \varepsilon^{-1} B B^{\mathrm{T}} P(\varepsilon) + \mathrm{I} = 0. \tag{7}$$

Here $\lim_{\varepsilon\to 0} P(\varepsilon) = P$, where P – solution of the Riccati equation for System (3) in the absence of an uncertainty matrix Λ.

In the second case, it is assumed that the matrix of parametric perturbations belongs to some parameterization set E for fixed matrices Q and R and is defined as

$$E\big(A + \Lambda(\mu)\big)_{\mathrm{m}} = {}^{1}\!/_{2}\, B R^{-1} B^{\mathrm{T}} P - {}^{1}\!/_{2}\, P^{-1} Q - mP, \tag{8}$$

where

$$m^{\mathrm{T}} = \begin{bmatrix} 0 & m_1 & m_2 & m_3 \\ -m_1 & 0 & m_4 & m_5 \\ -m_2 & -m_4 & 0 & m_6 \\ -m_3 & -m_5 & -m_6 & 0 \end{bmatrix} - \tag{9}$$

a skew-simetric matrix, the elements of which mi are determined by Kharitonov's theorem [3] on the robustness of linear dynamical systems. Formula (9) canonizes the matrix A + Λ(μ), and is later used to determine the compensation control *k* in the optimal stabilization law (5).

Denoting the matrix A + Λ(μ), which is canonized, by A*, we define the entire set of compensation regulators k according to [10] as

$$E(k)_{\beta\pi} = B^{*}(\mathrm{A}^{*} - {}^{1}\!/_{2}\, B R^{-1} B^{\mathrm{T}} P Q + (\beta)_k P + \mathrm{B}^{R}\pi, \tag{10}$$

where: B^{*} and A^{*} – matrices that are canonized; π – arbitrary matrix of the corresponding dimension; β – skew-simetric matrix; B^{R} – right divisor of zero of the maximum rank [10].

As a result, it can be argued that the regulator (5) synthesized in this way is robust on the parameterization set (8) and minimizes the functional (2). Approaches to the synthesis of the optimal law of stabilization of linear dynamical systems with uncertainty in parameters are considered rather difficult to implement and cannot provide the necessary dynamic indicators of transients of linear dynamical systems in stabilization modes. The modal robust stabilization proposed below lacks these disadvantages.

Taking into account (1) and (5), we write the equation of a closed optimal system in the form:

$$\dot{x}(t) = [\mathrm{A} + K]\bar{x}(\mathrm{t}) + [\Lambda + k]\bar{x}(\mathrm{t}). \tag{11}$$

Let the given constraints on the elements of the parametric uncertainty matrix Λ be related to the identification error, i.e.

$$|\lambda_{ij}| \leq \Lambda^{0}_{\mathrm{ij}}, \tag{12}$$

as well as quality indicators for transients for state variables in the form of:

$$|x_i(t)| \leq \sigma_i^0. \quad (13)$$

It is necessary to synthesize the control law (5) under conditions (2) and provide the specified quality indicators of transients (3) in the system of stabilization of linear dynamical systems with parametric uncertainty.

The modal robust stabilization proposed below is based on the principle of guaranteed dynamics [11]. This principle is based on the concept of permissibility, which uses primary indicators of the quality of transients, such as transition time, dynamic and static accuracy, as an assessment.

Let's write equation (11) in coordinate form

$$|\dot{x}_i(t)| = \sum_{J=1}^{n}(a_{ij} + K_{ij} + k_{ij} + \lambda_{ij})x_j(t). \quad (14)$$

According to [4,14], conditions (13) are met if

$$\int_0^t x_i(\tau)\dot{x}_i(\tau)d\tau \leq \int_0^t \sigma_i(\tau)\sigma_i(\tau)d\tau, \ \ i = 1,2 \dots, n; \ \ t \in [0, \infty]. \quad (15)$$

Substituting expression (14) into (15), we obtain

$$\int_0^t \left[\sum_{J=1}^{n}(a_{ij} + K_{ij} + k_{ij} + \lambda_{ij})x_j(\tau)\right]x_i(t)d\tau \leq \int_0^t \sigma_i(\tau)\sigma_i(\tau)d\tau, \quad (16)$$

where $i = 1,2 \dots, n; \ \ t \in [0, \infty]$.

Let's set $\sigma_i^0(t)$ as

$$\sigma_i^0(t) = \sigma_i^0 e^{\alpha t}, \quad (17)$$

where σ_i^0 are chosen as estimates of the maximum possible deviations xi(t) at the initial time, and α is determined from the condition of a given degree of attenuation βi of the transition process (fig.1) and the same for all state variables [13], i.e.

$$e^{\alpha t_k} \leq \beta_i, \quad (18)$$

where $\alpha \leq 0, t_k -$ the specified transition time.

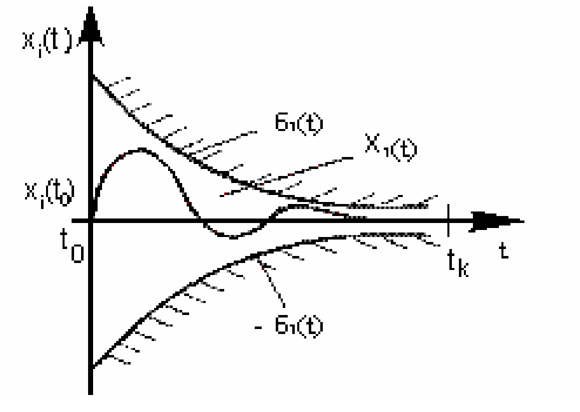

Figure 1. Limits of the acceptable range for changing the i-th parameter

These dynamic indicators of transients are provided by the appropriate choice of the spectrum of roots of a closed optimal system in accordance with the work [12].

Given (13), (17), and (18), equation (16) takes the form:

$$\int_0^t \left[\sum_{J=1}^{n}(a_{ij} + K_{ij} + k_{ij} + \lambda_{ij}^0)\sigma_j^0\right]\sigma_i^0 e^{2\alpha\tau}d\tau \leq \int_0^t \alpha(\sigma_i^0)^2 d\tau, \quad (19)$$

where $i = 1,2 \dots, n; \ \ t \in [0, \infty]$.

Integrating inequality (19) on the interval t ∈ [0,∞], we obtain a system of linear algebraic inequalities

$$2\left[\sum_{J=1}^{n}(a_{ij}+K_{ij}+k_{ij}+\lambda_{ij}^{0})\sigma_{j}^{0}\right]\sigma_{i}^{0}e^{2\alpha\tau}\leq\sigma_{i}^{0}, i=1,2\dots,n. \quad (20)$$

Hence, it is determined that a set of k_{ij} values satisfying the system of inequalities (20) ensures the stability of system (1) to parametric perturbations based on the control law (5) obtained by the above modal synthesis procedure. One of the options for practical determination of the values of k_{ij} compensation regulators is the solution of the system of inequalities (20) at the boundaries of the permissible regions $+\alpha\sigma_i^0$ and $-\alpha\sigma_i^0$ using well-known numerical methods [14].

Conclusion

This article proposes a modal approach to the synthesis of robust systems for optimal stabilization of the program flight path of a UAV under conditions of uncertainty of its state parameters. The synthesis of a modal robust controller is implemented using the principle of guaranteed dynamics, which allows ensuring not only stability, but also the required quality of control. Its essence is that with possible permissible variations in the parameters of the state of the UAV, transients in the stabilization system are guaranteed to remain within the specified permissible areas. The boundaries of these regions are set by the corresponding location of the roots of the closed optimal stabilization system and a given permissible error in identifying the parameters of the UAV state.

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