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A. Stenin, V. Pasko, M. Soldatova, I. Drozdovich, S. Stenin

SYNERGETIC SYNTHESIS OF OPTIMAL CONTROL LAWS OF NONLINEAR DYNAMIC OBJECTS

Abstract: The synergetic synthesis of optimal stabilizing control laws for one class of nonlinear dynamic objects proposed in the article is based on the joint use of the well-known ACOR method and a specially designed synergetic functional. Synergetic synthesis of optimal stabilizing control laws based on the ACOR method and a specially designed synergetic functional that meets the conditions formulated in the article allows, on the one hand, to ensure asymptotic stability and, on the other hand, to ensure the specified dynamic properties of transient processes for one class of nonlinear dynamic systems. The practical implementation of this approach is demonstrated using the example of the synergetic synthesis of the law of optimal stabilization given in the article.

*Keywords***:** ACOR, nonlinear object, Lyapunov functions, synergetic approach, accompanying functionality, stability, specified dynamic properties

Introduction

The nonlinearity of a fairly large class of real technical systems is a significant obstacle in the development of control systems for them. In particular, the synthesis of optimal control laws for nonlinear dynamic objects is a very complex task, which often cannot be solved in analytical form [1, 2]. Primarily due to the fact that the principle of superposition is not observed for nonlinear differential equations. In addition, in problems of synthesizing optimal control laws for linear dynamic objects, the structure of the control law is predetermined (ACOR problem), while for nonlinear dynamic objects the structure of optimal control laws is generally unknown. At the same time, the ACOR theory can be used as a guiding structural-parametric concept in the development of methods for synthesizing optimal control laws for nonlinear dynamic objects. Thus, the task of synthesizing optimal control laws for nonlinear dynamic objects comes down to finding a solution to the Bellman equation with respect to some generating function that defines this optimal control law. The work [2] shows the connection between the Lyapunov functions in problems of asymptotic stability of the motion of a nonlinear dynamic object and the Bellman equation, which is satisfied by a certain set of generating functions. The necessary solutions can be selected by base the Lyapunov method [3]. Moreover, these solutions will always exist due to the stability postulate N.G. Chetaev [4]. It follows that the synthesis of optimal control laws for nonlinear dynamic objects can be based on the construction of asymptotically stable motions described by a certain set of differential equations. This general concept received significant development in the works [3,5,6.

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Statement of the problem

Let the dynamics of a nonlinear dynamic object be described by a system of nonlinear differential equations of the form:

$$\dot{x}_i(t) = f_i(x_1 \dots x_n, u_1 \dots u_m), i = 1, \dots, n; m \leq n. \quad (1)$$

The problem of synthesizing an optimal control law can be formulated as follows: to ensure asymptotic stability of the perturbed motion of a nonlinear object (1) in a certain region of phase space or its asymptotic stability as a whole, it is necessary to select among the set of possible control laws $u(x_1, \dots, x_n)$, some optimal set or one optimal control law.

Based on the Barbashin-Krasovsky theorem on asymptotic stability in general [7], an approach to obtaining sufficient conditions for stabilizability was formed, which consists of the following [3].

A positive definite Lyapunov function $V(x_1, \dots, x_n) > 0$ is introduced and its total time derivative is found:

$$\dot{V}(t) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x_1 \dots x_n, u_1 \dots u_m) + \frac{\partial V}{\partial t}. \quad (2)$$

The task is to select a control $u(x_1, \dots, x_n)$, for which the following condition is satisfied:

$$V(t) \leq -W(x_1 \dots x_n, t), \quad (3)$$

where $W(x_1 \dots x_n, t)$ is a given positive definite function. Taking into account (3), expression (2) will take the form:

$$S(x_1 \dots x_n, u, t) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x_1 \dots x_n, u_1 \dots u_m) + W(x_1 \dots x_n, t) + \frac{\partial V}{\partial t} \leq 0. \quad (4)$$

Obtaining an analytical solution of the control laws $u_{st}(x_1 \dots x_n)$, ensuring the fulfillment of condition (4), is possible only under certain conditions. This explains the problem of synthesizing stabilizing control laws for fairly general classes of nonlinear objects.

Analysis of existing solutions

In addition to stability, as one of the set of requirements for the dynamic properties of the synthesized optimal system, it is equally important to provide requirements for the quality of control processes (class of ACOR problems), which in the theory of optimal systems is usually assessed by integral quality criteria of the form:

$$I = \int_0^\infty W(x_1 \dots x_n, u_1 \dots u_m) dt, \quad (5)$$

where $W(x_1 \dots x_n, u_1 \dots u_m)$ is some non-negative function on the trajectories of the original object (1).

At present, the solution of ACOR problems for linear objects has been studied quite well and a number of works are devoted to them. In the nonlinear ACOR theory, the solution to these problems is based on the optimal stabilization theorem [7], which establishes a connection between the problems of stability and optimality of control systems. This relationship can be written in analytical form in the following form:

$$\dot{V}_0(t) = -W\left(x_1 \dots x_n, u_{jopt}\right), j = 1,2, \dots, m. \quad (6)$$

Equation (6) says that the use of optimal Lyapunov functions $V_0(x_1 \dots x_n)$ allows us to select from the entire set of possible controls those that provide both asymptotic stability and optimality of the system according to the corresponding quality criterion.

Depending on the type of integrand functions $W\left(x_1 \dots x_n, u_j\right)$ used in criterion (5), we have:

$$min\int_0^\infty W\left(x_1 \dots x_n, u_{jopt}\right) = V_0(x_1 \dots x_n) \quad (7)$$

and during optimization we can obtain various dynamic properties of the system.

The ability to search for solutions to equation (6), due to the initial equations of the object (1), determines success in the problem of synthesizing an optimal system. In this regard, it should be noted that the difficulties of synthesizing nonlinear optimal systems come down to the rational choice in (7) of the integrand functions $W(x_1 \dots x_n, u_1 \dots u_m)$, reflecting the requirements for the quality of the dynamics of systems, as well as to the search for solutions to equation (6). An insufficiently justified choice of function W, although it can lead to stable motion of the system, however, stabilizing controls built on its basis may turn out to be of little use for practical implementation.

Let us specify the AKOR procedure for nonlinear objects (1) with linearly incoming controls:

$$\dot{x}_i(t) = f_i(x_1 \dots x_n) + \sum_{j=1}^{m} G_{ij}(x_1 \dots x_n)u_j, i = 1,2, \dots, n, m \leq n. \quad (8)$$

For this class of objects, the ACOR procedure was quite well developed by A.M. Letov and R. Kalman. The ACOR Letov-Kalman method [8,9] considers the optimization of objects (8), for which criterion (5) has the following form:

$$I = \frac{1}{2}\int_0^\infty \left(\sum_{i,k=1}^{n} \beta_{ik}x_i x_k + \sum_{j=1}^{m} m_j^2 u_j^2\right)dt. \quad (9)$$

In this case, the control that provides a minimum to criterion (9) will have the form:

$$u_j = -\frac{1}{m_j^2}\sum_{i=1}^{n} G_{ij}(x_1 \dots x_n)\frac{\partial V_0}{\partial x_i}, \quad (10)$$

where $V_0(x_1 \dots x_n)$ is the solution to the equation

$$\sum_{i=1}^{n} \frac{\partial V_0}{\partial x_i} f_i - \frac{1}{2m_j^2}(G_{ij}\frac{\partial V_0}{\partial x_i})^2 = -\frac{1}{2}\sum_{i,k=1}^{n} \beta_{ik}x_i x_k. \quad (11)$$

In applied terms, the task of synthesizing an optimal control law in the form of expression (10) is reduced to searching for a forced solution $V_0(x_1 \dots x_n)$ of equation (11), which is a nonlinear partial differential equation. Unfortunately, there are no methods for analytical or even numerical solution of this equation, although, as a mathematical procedure, equations (5) - (11) have been known in the literature for a long time [2]. The development of methods for solving these equations would provide significant progress in solving the nonlinear ACOR problem.

Let us move on to the method of A.A. Krasovsky [10], which for optimization of nonlinear objects (1) involves the use of the generalized work criterion:

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$$I = \frac{1}{2}\int_0^\infty \left[\sum_{i,k=1}^{n} \beta_{ik} x_i x_k + \sum_{j=1}^{m} m_j^2 u_j^2 + \sum_{j=1}^{m} (m_j \sum_{i=1}^{n} \mathrm{B}_{ij} \frac{\partial V_0}{\partial x_i})^2\right]. \qquad (12)$$

In this case, the optimal control law that ensures the minimum of criterion (12) is determined by the expression

$$u_j = -\frac{1}{m_j^2} \sum_{i=1}^{n} \mathrm{B}_{ij} \frac{\partial V_0}{\partial x_i}, \qquad (13)$$

where the function$V_0(x_1 \dots x_n)$ in (13) is a forced solution to the equation

$$\frac{\partial V_0}{\partial x_i} + \sum_{i=1}^{n} \frac{\partial V_0}{\partial x_i} f_i = -\frac{1}{2} \sum_{i,k=1}^{n} \beta_{ik} x_i x_k. \qquad (14)$$

A characteristic feature of equation (14) is that its left side can be considered as the time derivative of the function $\dot{V}_0(t)$ for $u_j = 0$, i.e. V_0 is the Lyapunov function, and equation (14) is the Lyapunov equation for the uncontrolled object. This raises the problem with the direct application of A.A. Krasovsky's method for nonlinear objects, which, in the absence of controls *(*$u_j = 0$*)*, must be stable or pre-stabilized using a separate system. In this regard, in [9] a procedure for applying this method for unstable and neutral objects is proposed, based on the transformation of the original differential equations (1) and the functional (12) to special non-stationary forms.

The advantage of A.A. Krasovsky's method with the generalized work criterion (12) is that equation (14), unlike equation (11), is already a linear partial differential equation. This allows, for equation (14) with different boundary conditions, to develop approximate methods for solving it based on power series by repeatedly applying a certain operator or relying on the method of characteristics [11]. Also of undoubted interest is the use of predictive models of management processes [12].

The method of A.A. Krasovsky is actually the only method in modern control theory that allows for a generalized combined synthesis, when not only the current formation of the control law is realized, but also current identification can occur, which in the literature is defined by the term "dual control".

Recently, the method of analytical design of aggregated controllers (ACAR) proposed in [13] for the synthesis of optimal stabilizing laws for the control of nonlinear dynamic objects has attracted great interest. The ACAR method is based on the introduction of accompanying functionals of the form:

$$I_\Sigma = \int_0^\infty F\left(\psi, \dot{\psi}\right) dt, \qquad (15)$$

where $F(\psi, \dot{\psi})$ is a function continuously differentiable with respect to its arguments; $\psi(x_1 \dots x_n)$ is an aggregated macrovariable, which is some arbitrary differentiable or piecewise continuous function of phase coordinates $(x_1 \dots x_n)$, and $\psi(0,\dots,0) = 0$.

Based on the ACAR method, it is possible to determine in the phase space of the nonlinear object under consideration a certain set of points to which all nearby motion trajectories are attracted. Such a set of points is usually called an attractor [13].

For a nonlinear object of the form

$$\dot{x}_i(t) = f_i(x_1 \dots x_n), i = \overline{1, n-1},$$
$$\dot{x}_n = f_n(x_1 \dots x_n) + u. \quad (16)$$

for scalar control, it is proposed to choose the following quadratic form as an integrand function [5,7,13]:

$$F(\psi, \dot{\psi}) = m^2\varphi^2(\psi) + c^2\dot{\psi}^2(t). \quad (17)$$

In this case, the functions φ(ψ) must satisfy the following conditions:

1. uniqueness, continuity and differentiability for all values ψ;
2. $\varphi(0) = 0$;
3. $\varphi(\psi)\psi \geq 0$ for any $\psi \neq 0$.

In other words, the functions *φ(ψ)* in this case will have the same sign as *ψ*, and they vanish only on the manifold *ψ*=0.

Having determined the total derivative of the function *ψ*

$$\frac{\partial\psi}{\partial t} = \sum_{k=1}^{n} \frac{\partial\psi(x_1 \dots x_n)}{\partial x_k} \dot{x}_k(t).$$

and substituting it and the right-hand sides of equation (16) into (17), we obtain the integral criterion (15) in the following form:

$$I_\Sigma = \int_0^\infty \left[m^2\varphi^2(\psi) + c^2\left(\sum_{k=1}^{n} \frac{\partial\psi}{\partial x_k} f_k + \frac{\partial\psi}{\partial x_n} u\right)^2\right] dt.$$

The main difficulty in the practical use of the ACAR method for optimizing nonlinear dynamic systems remains the correct choice of accompanying functionals.

It should be noted that for linear dynamic systems, the use of the ACOR method and the ACAR method gives identical results.

Let us demonstrate this fact using the example of a linear dynamic system of the form:

$$\dot{x}(t) + ax = u, \quad (18)$$

for which we construct a quality criterion based on the accompanying functional (15) in the form:

$$I_1 = \int_0^\infty \left[m^2\psi^2 + c^2\dot{\psi}^2(t)\right] dt. \quad (19)$$

Moreover

$$\dot{\psi}(t) = \frac{\partial\psi}{\partial x} \dot{x}(t). \quad (20)$$

Let us determine the derivative of the macrovariable *ψ* on solutions of the object (18), for which we substitute the derivative of the variable $\dot{x}(t$ from (18) into expression (20), i.e.:

$$\dot{\psi}(t) = \frac{\partial\psi}{\partial x}(u - ax). \quad (21)$$

Let us now substitute expression (21) into functional (19):

$$I_1 = \int_0^\infty \left[m^2\psi^2(x) + c^2(a^2x^2 - 2axu)\left(\frac{\partial\psi}{\partial x}\right)^2 + c^2\left(\frac{\partial\psi}{\partial x}\right)^2 u^2\right] dt. \quad (22)$$

Let us $\psi = x$, тогда (22) примет вид:

$$I_1 = \int_0^\infty [(m^2 + a^2c^2)x^2 - 2ac^2xu + c^2u^2] dt \quad (23)$$

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Next, using the Lagrange function $\lambda(t)$, we write, taking into account the integrand in (23), the Lagrangian in the form

$$L_1 = (m^2 + a^2c^2)x^2 - 2ac^2xu + c^2u^2 + \lambda(\dot{x} + ax - u). \quad (24)$$

For the Lagrangian (24), the Euler-Lagrange equations take the form:

$$\frac{\partial L_1}{\partial x} - \frac{d}{dt}\left(\frac{\partial L_1}{\partial \dot{x}}\right) = 2(m^2 + a^2c^2)x - 2ac^2u + \lambda a - \dot{\lambda}(t),$$

$$\frac{\partial L_1}{\partial u} - \frac{d}{dt}\left(\frac{\partial L_1}{\partial \dot{u}}\right) = 2c^2u - 2ac^2x - \lambda.$$

It is easy to show that for a stable system the optimal control is

$$u_0 = -\left(\frac{m}{c} - a\right)x$$

completely coincides with the result of the standard ACOR procedure for object (18) and quadratic criterion (9) of the form

$$I_1 = \int [(m^2 - a^2c^2)x^2 + c^2u^2]dt.$$

The problem of correctly selecting accompanying functionals can be avoided by using the method proposed below for the synergetic synthesis of optimal stabilizing control laws for one class of nonlinear dynamic objects.

Synergetic synthesis of optimal stabilizing control laws for one class of nonlinear dynamic objects

The synergetic synthesis procedure proposed below is based on the use of a functional that, on the one hand, fully corresponds to the main functional of the ACOR problem, and on the other hand, corresponds to the solution of the original optimization problem in the form of a quadratic positive definite form. The above actually corresponds to the basic principle of synergetic approach - the principle of subordination, on which the theory of self-organization of nonlinear dynamic systems is built [11,13].

Let us formulate the problem statement and show the general procedure for synergetic synthesis.

Let the mathematical model of a nonlinear dynamic object be described by a system of differential equations of the form:

$$\dot{\bar{x}}(t) = A(x)\bar{x}(t) + Bu(t), \quad (25)$$

where $\bar{x} = (x_1, x_2, ..., x_n)^{\mathrm{T}}$ is the vector of deviation of variables from the given state of a nonlinear dynamic object; u-scalar control; A-matrix of coefficients dimensions n x n with, some of which are nonlinear functions of environmental state variables; B is a column vector of dimension (n x 1)

Let us set the management quality functional according to the ACOR problem in the form:

$$V(t) = min_u \int_0^\infty [\bar{x}^T Q \bar{x} + u^2]dt, \quad (26)$$

where $Q = \{q_{ij}\}$ - diagonal matrix of dimensions n x n.

It is necessary to find a control $u^{opt}(\bar{x})$ that transfers system (25) from some initial state to zero and minimizes functional (26).

In such a setting, it is inappropriate to use the standard ACOR procedure, because its solution is significantly complicated by the fact that the vector-matrix Riccati equation will include a number of nonlinear coefficients a_{ij}.

Application of the dynamic programming method to system (25) taking into account (26) leads to the following Bellman functional equation (5.7) [1,2]:

$$-\frac{\partial V}{\partial t} = \min{}_u \left[\bar{x}(t)^T Q\bar{x}(t) + u^2 + \frac{\partial V}{\partial \bar{x}}(A(x)\bar{x}(t) + Bu)\right] \tag{27}$$

Obviously, the control minimum in (27) is achieved under the condition

$$u^{opt}(\bar{x}) = -\frac{1}{2}\frac{\partial V}{\partial \bar{x}}B \tag{28}$$

To determine the desired optimal control (28), it is necessary to form a functional in the state space in the form of a Lyapunov function $V(\bar{x})$, which satisfies equation (27) in partial derivatives.

Based on the theorem on the asymptotic stability of nonlinear dynamic objects [3], we represent the function $V(\bar{x})$ in the form of a positive definite quadratic form of the form [3].

$$V = \bar{x}^T N\bar{x} \tag{29}$$

where N is a positive definite symmetric matrix of dimension n x n.

Hence, the derivatives of the function V with respect to the variables xi are determined from expression (29) as

$$\begin{gathered}\frac{\partial V}{\partial x_1} = 2v_{11}x_1 + v_{12}x_2 + \cdots + v_{1n}x_n \\ \cdots \\ \frac{\partial V}{\partial x_n} = v_{n1}x_1 + v_{n2}x_2 + \cdots + 2v_{nn}x_n\end{gathered} \tag{30}$$

At the same time, from (30) it also follows that

$$-\frac{\partial V}{\partial t} = -\sum_{i=1}^{n} \upsilon_{ii}x_i x_i - 2\sum_{i,j=1}^{n} \upsilon_{ij}x_i x_j;\ (i \neq j) \tag{31}$$

Equating the right-hand sides of expressions (31) and (27), taking into account (30), we obtain an equation in which we can group expressions with respect to x_i^2 and $x_i x_j$, since in general they are not equal to zero. In addition, since the stationarity condition [14] is satisfied for system (25) and functional (26), therefore, we can set $\dot{v}_{ij} = 0 (i,j = 1,2,\dots,n)$. The expressions grouped in this way represent a system of nonlinear algebraic equations of dimension $n(n+1)/2$ with respect to the unknowns υ_{ij}. The solution of this system of equations gives the desired optimal control (28).

Synthesis example

As a mathematical model, we use a system of differential equations of the following form:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + Bu(t), \tag{32}$$

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where $\bar{x} = (x_1, x_2)$-state vector;u - scalar control; matrix A has the following form

$$A = \begin{bmatrix} 0 & 1 \\ 0 & \cos(x_1) \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We set the main functional of control quality in the form (26), i.e.

$$V(t) = min \int_0^\infty [q_{11}x_1^2(t) + q_{22}x_2^2(t) + u^2(t)]dt. \quad (33)$$

It is necessary to find a control $u^{opt}(t)$, that minimizes functional (26).

As stated above, applying the dynamic programming method to system (32) leads to a functional Bellman equation of the form (27). In this case, the minimum in our problem is achieved under the condition

$$u^{opt}(t) = -\frac{1}{2}\frac{\partial V}{\partial \bar{x}}B = -\frac{1}{2}\frac{\partial V}{\partial x_2}. \quad (34)$$

Next, to determine the desired optimal control (34), we form the Lyapunov function $V(\bar{x})$, which satisfies the partial differential equation

$$-\frac{\partial V}{\partial t} = q_{11}x_1^2(t) + q_{22}x_2^2(t) - \frac{\partial V}{\partial \bar{x}}A\bar{x} - \frac{1}{4}\left(\frac{\partial V}{\partial x_2}\right)^2 = 0. \quad (35)$$

Based on (30), we represent the Lyapunov function in the form

$$V = v_{11}x_1^2 + v_{12}x_1x_2 + v_{22}x_2^2. \quad (36)$$

Then we can write

$$\begin{cases} \frac{\partial V}{\partial x_1} = 2v_{11}x_1 + v_{12}x_2 \\ \frac{\partial V}{\partial x_2} = v_{12}x_1 + 2v_{22}x_2 \end{cases}. \quad (37)$$

Considering that from (37) it follows

$$-\frac{\partial V}{\partial t} = -\frac{\partial v_{11}}{\partial t}x_1^2 - \frac{\partial v_{12}}{\partial t}x_1x_2 - \frac{\partial v_{22}}{\partial t}x_2^2. \quad (38)$$

Making substitutions (37) and (38) into (35) we get

$$-\dot{v}_{11}x_1^2 - \dot{v}_{12}x_1x_2 - \dot{v}_{22}x_2^2 = q_{11}x_1^2 + q_{22}x_2^2 + (2v_{11}x_1 + v_{12}x_2)x_2; \quad (39)$$

$$-\frac{1}{4}(v_{12}x_1 + 2v_{22}x_2)^2 = 0.$$

Let's group the expressions with respect to x_1 and x_2, because in the general case they are not equal to zero. We get

$$\begin{cases} \dot{v}_{11} = -q_{11} + \frac{1}{4}v_{12}^2 \\ \dot{v}_{22} = -q_{22}v_{12} + v_{22}^2 \\ \dot{v}_{12} = -2v_{11} + v_{12}v_{22} \end{cases}. \quad (40)$$

Since system (32) is stationary in time, then, based on Lyapunov's theorem, in equation (40) we should put $\dot{v}_{ij} = 0 (i,j = 1,2)$.

From here

$$\begin{cases} v_{12}^2 - 4q_{11} = 0 \\ v_{22}^2 - q_{22} - v_{12} = 0. \\ v_{12}v_{22} - 2v_{11} \end{cases} \quad (41)$$

Solving system (41), we obtain

$$v_{12} = 4\sqrt{q_{11}};\ v_{22} = \sqrt{q_{22} + 4\sqrt{q_{11}}};v_{11} = 2\sqrt{q_{11}q_{22} + 4q_{11}}.$$

Taking (34) into account, we finally obtain

$$u^{opt}(t) = -\frac{1}{2}b_1(2v_{11}x_1 + v_{12}x_2) = -2b_1\sqrt{q_{11}q_{22} + 4q_{11}}x_1 - 2b_1\sqrt{q_{11}}x_2.$$

Conclusion

Synergetic synthesis of optimal stabilizing control laws based on the ACOR method and a specially designed synergetic functional that meets the conditions formulated in the article allows, on the one hand, to ensure asymptotic stability and, on the other hand, to ensure the specified dynamic properties of transient processes for one class of nonlinear dynamic systems. The practical implementation of this approach is demonstrated using the example of the synergetic synthesis of the law of optimal stabilization given in the article.

REFERENCES

1. Letov A.M. Some unsolved problems in the theory of automatic control / A.M. Letov – M.: Nauka, 1966. – 256c.
2. Krasovsky A.A. Automatic flight control systems and their analytical design / A.A. Krasovsky - M.: Nauka, 1973. - 560c.
3. Kuntsevich V.M. Synthesis of automatic control systems using Lyapunov functions / V.M. Kuntsevich, M.M.Lychak - M.: Nauka, 1977. - 400p.
4. Chetaev N.G. Stability of movement. Works on analytical mechanics / N.G. Chetaev – M.: Nauka, 1965. – 176c.
5. N.Rush, P. Abets, M. Lalua. Direct Lyapunov method in stability theory. - Moscow: Mir, 1980.- 300 p.
6. Furasov V.D. Stability of movement, assessments and stabilization / V.D. Furasov – M.: Nauka 1977. – 248c.
7. Barbashin E.A. Lyapunov functions/ E.A. Barbashin – M.: Nauka, 1970. – 240c.
8. Letov A.M. Analytical design of regulators // Automation and telemechanics. Vol. 21, No. 4, 1960. - P. 436-441.
9. Kalman R. Contribution to the theory of optimal control/ R. Kalman // Bul.Soc.Mech.Mat. –Vol.12, No.2, 1960. – P. 102-119.
10. Krasovsky N.N. Problems of stabilization of controlled movements / N.N. Krasovsky – M.: Nauka, 1987. – 456 p.
11. Haken. G. Synergetics. Hierarchy of instability in self-organizing systems and devices / G. Haken. – M.: Mir, 1980. – 404p.
12. Camacho E., Bordons C. Model predictive control / Camacho E., Bordons C. London: Springer-Verlag, 2007. - 423p.
13. Kolesnikov A.A. Modern applied management theory: Synergetic approach to management theory, part II / Ed. A.A. Kolesnikova. Taganrog: TRTU Publishing House, 2000. - 559 p.

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