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A. Stenin, V. Pasko, M. Soldatova, I. Drozdovich

OPTIMIZATION OF ELECTRICITY COSTS IN RESIDENTIAL HEAT SUPPLY SYSTEMS

*Abstract***.** Currently, there is a large number of objects related to recirculation of material flows, mixing of reagents of liquid and gaseous media, etc. Among them are heat supply systems of residential buildings. Their dynamics are described by differential equations with a deviating argument. Optimizing the operation of such objects using known methods is quite difficult. This paper proposes a dummy variable method for reducing the original system of differential equations with a deviating argument to a system of ordinary differential equations and transfer equations for which known optimization methods can be applied. The method is generalized to systems with a non-stationary deviating argument and to systems with several processes having an aftereffect. The practical use of the dummy variable method is shown on the example of solving the problem of optimal control of the heat exchange process in a private residential building.

Keywords: technological process, aftereffect, dummy variable, equivalent scheme of equations, residential building heating circuit, optimal algorithm

Introduction

Numerous processes occurring in living nature, economic systems and technical devices are characterized by the fact that their behavior depends on the prehistory of their course at a certain time interval, i.e. by the presence of an aftereffect [1,2]. Such systems with aftereffects are described by differential equations with a deviating argument. Recently, much attention is paid to the solution of problems of optimal control of systems with aftereffects. This is due to the emergence of a large number of technological processes associated with recirculation of material flows, reagent mixing of liquid and gaseous media, etc. A consequence of this is a large number of theoretical investigations of the qualitative properties of dynamical systems with an aftereffect. The results obtained are widely used in automatic control, mechanics, technology, economics, medicine and other industries. However, the study of systems with after-action is associated with significant difficulties, as a consequence of which the exact analytical solution of optimal control problems can be obtained only in exceptional cases. Moreover, along with the usual difficulties for finite-dimensional problems, the consideration of controllable systems with aftereffects is associated with a number of specific difficulties due to the fact that the phase space of these systems is infinite dimensional as a rule. Overcoming such difficulties led to the development of various methods for solving control problems oriented to certain classes of systems with aftereffects. Recirculation processes associated with the mixing of water of different temperatures in the batteries of the heating circuit are inherent in most of the heat supply systems of residential buildings. Hence, the minimization of energy costs for establishing and maintaining the optimal temperature regime is an urgent task, for the solution of which the dummy variable method is proposed in this article and the optimal control algorithm is implemented.

Analysis of existing approaches

A number of monographs [3-7] and many articles are devoted to the problem of optimal control of systems with aftereffects. However, in spite of a large number of publications in this field, the methods proposed therein allow one to solve only partial problems with respect to specific dynamical systems with aftereffects. Thus, the paper [8] obtains the necessary condition of optimality of the equality type for systems with delays and integral control limitations, the papers [9,10] investigate a linear dynamic system stability dependence on the form of its description, the papers [11,12] discuss a robust control synthesis in the presence of an after-action, the papers [13,14] develop methods for optimal stabilization of a neutral dynamic system with a given spectrum. A number of papers have shown that the optimal control problem for some simple systems with constant time lag can be solved on the basis of known optimization methods, in particular, on the basis of the maximum principle and the phase plane method [15,16]. Real dynamic systems are described, as a rule, by higher-order equations. For higher-order systems, the synthesis of an optimal control causes fundamental difficulties in obtaining an exact or approximate solution. This leads to the necessity of transition to systems equivalent to the initial systems with an aftereffect. A dummy variable method is proposed below that allows one to go from the initial system with an aftereffect to an equivalent system without an aftereffect.

Problem statement

For a system described by a differential-difference equation of the form

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t),\boldsymbol{x}(t-\tau),\boldsymbol{u}(t)) \tag{1}$$

with an initial condition

$$\boldsymbol{x}(t) = \boldsymbol{x}_0(t),\ \mathrm{t} \in [-\tau\,,0] \tag{2}$$

it is necessary to construct an equivalent system without delay τ.

Method of dummy variables

Let us move from the notation (1) and (2) to a system of ordinary differential equations and partial differential equations and transfer partial differential equations [4]:

$$\frac{dx(t)}{dt} = f(\boldsymbol{x}(t), \boldsymbol{z}(\tau), \boldsymbol{u}(t), t), \tag{3}$$

$$\frac{dy(t,\theta)}{dt} + \frac{dy(t,\theta)}{d\theta} = 0 \tag{4}$$

with initial and boundary conditions:

$$\boldsymbol{x}(0) = \boldsymbol{x_0}(0);\ \boldsymbol{y}(0,\theta) = \boldsymbol{y_0}(\theta), \theta \in [0,\tau];$$
$$\boldsymbol{z}(t) = \boldsymbol{y}(t,\tau), t \in [0,T]. \quad (5)$$

Here $\boldsymbol{y_0}(\theta) = \boldsymbol{x_0}(-\theta)$, T – control time.

Substituting the solution of $\boldsymbol{y}(t,\tau)$ equation (4) into relation (3), it is easy to see the equivalence of these representations. The variable θ and vector $\boldsymbol{y}(t,\tau)$ play the role of a dummy variables and a state vector for the case when there is no pure delay in the system defined by equation (1).

The advantages of this definition are obvious since equation (4) can be applied to describe more general processes of systems with deviating arguments. To do this, it is sufficient to introduce some multiplier into this equation $g(t,\theta)$ and modify the right-hand side accordingly to obtain an equation of the form:

$$\frac{dy(t,\theta)}{dt} + g(t,\theta)\frac{dy(t,\theta)}{d\theta} = \boldsymbol{h}(\boldsymbol{y}(t,\theta), \boldsymbol{w}(t,\theta), t, \theta),\ t \in [0,T],\ \theta \in [0,\tau] \quad (6)$$

with the same initial and boundary conditions (5) and keep the same system of ordinary differential equations (3). Equation (6) is known as the linear transfer equation [21]. Here is an $\boldsymbol{x}(t)$ – n-dimensional vector of state of the system with lumped parameters, $\boldsymbol{y}(t,\theta)$ – an n-dimensional vector of state of the process with pure delay, $\boldsymbol{z}(t) = \boldsymbol{y}(t,\tau)$ an n-dimensional output which is at the same time an input variable for the system (3), $\boldsymbol{u}(t)$ – r-dimensional vector of control of the system with lumped parameters, and $\boldsymbol{w}(t,\theta)$ – s-dimensional vector of control with pure delay (Fig. 1).

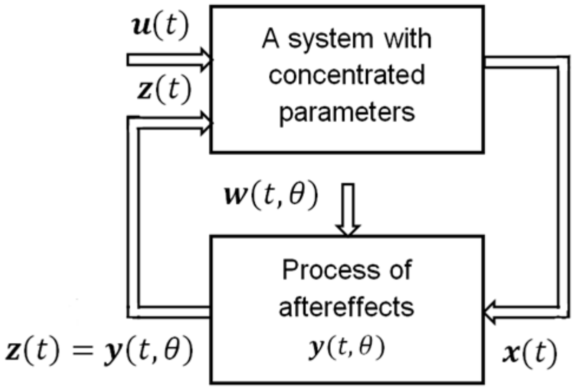

Figure 1. Equivalent scheme of the system with aftereffects

It is assumed that the functions $\boldsymbol{f}(.)$ and $\boldsymbol{h}(.)$ have continuous second derivatives on all arguments and satisfy Lipschitz conditions. The function $g(t,\theta)$ is continuous and, moreover, $g(t,0) > 0, g(t,\tau) > 0$ the derivative of the vector function $\boldsymbol{f}(.)$ over the vector variable $\boldsymbol{z}(t)$ does not depend explicitly on the control vector $\boldsymbol{u}(t)$. The admissible piecewise continuous controls $\boldsymbol{u}(t)$ and $\boldsymbol{w}(t,\theta)$ take values in given convex regions $\boldsymbol{u} \in U, \boldsymbol{w} \in W$. It can be shown that for given vectors of admissible controls $\boldsymbol{u}(t)$ and $\boldsymbol{w}(t,\theta)$ trajectories $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t,\theta)$ are uniquely determined by their initial and final conditions.

Using the dummy variable method described above, the systems with non-stationary lag given by the differential-difference equation:

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{x}\big(t-\tau_1(t)\big), \boldsymbol{x}\big(t-\tau_2(t)\big), \dots, \boldsymbol{x}\big(t-\tau_m(t)\big), \boldsymbol{u}(t)\right).$$

It is easy enough to be reduced to the form

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{z}_1, \boldsymbol{z}_2, \dots \boldsymbol{z}_m, \boldsymbol{u}(t), t),$$

$$\frac{d\boldsymbol{y_j}(t,\theta)}{dt} + g_j(t,\theta)\frac{dy_j(t,\theta_j)}{d\theta_j} = \boldsymbol{h}_j\,(y_j\big(t,\theta_j\big), \boldsymbol{w}_j\big(t,\theta_j\big), t, \theta_j))$$

at $j = 1,2,\dots,m; t \in [0,T]; \theta_j \epsilon [0,\tau_j]$ and initial and boundary conditions

$$\boldsymbol{x}(0) = \boldsymbol{x}_0(0);\ \boldsymbol{y_j}(0,\theta_j) = \boldsymbol{y_{j0}}\big(\theta_j\,\big), \theta_j \in [0,\tau_j],$$

$$\boldsymbol{y_j}(t,0) = \boldsymbol{x}(t), t \in [0,T], \tag{9}$$

$$\boldsymbol{z}(t) = \boldsymbol{y_j}(t,\tau),\ t \in\ [0,T],\ \theta_j \in [0,\tau_J]$$

by selecting the functions accordingly $g_j(t,\theta)$ and $\boldsymbol{h}_j\ (t,\theta)$

$$g_j(t,\theta_j(t)) =\ (\tau_j(t) - \theta_j(t)\frac{d\tau_j(t)}{dt})/\tau_j(t);\ \ \boldsymbol{h}_j\ \left(t,\theta_j(t)\right) = 0. \tag{10}$$

Equivalence of relations (7) – (10) follows from the conditions

$$\boldsymbol{y}_j(0,\theta_j) = (t - \theta_j\tau_j(t)), j = 1,2,\dots,m.$$

Optimal control of the heat transfer process in a private residential house

Recently, in Ukraine, special attention is paid to the solution of energy saving tasks in the communal economy. In particular, the problem of synthesis of optimal control in an autonomous system of hot water supply and heating of a private house is considered below. The standard scheme of this system is shown in Fig. 2.

It is known that the process of heat transfer in the circuit of heating and hot water supply of a private residential building is accompanied by the process of mixing water, that is, the process of recirculation [17]. Therefore, this process can be described by a system of differential equations with a delay argument, which is a simplified model of the Navier-Stokes equations [18,19]. The structural scheme of the heat exchange process is shown in Fig. 3.

In the autonomous heat supply system shown in Fig. 2, two main modes of operation can be distinguished:

- transition mode;
- stationary mode.

Transition mode. In this case, the output of the heating circuit to the nominal (stationary) mode of operation is considered in the presence of the specified temperature regime in the accumulator tank, and, therefore, also in the hot water supply circuit. Since the temperature in the storage tank is closely related to the heating circuit, the task is to set the specified value of the water temperature both in it and in the heating circuit. This can happen in different ways:

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– through the coil of the primary coolant in the form of hot liquid from the boiler (Fig.3) or from a geothermal source;

– through the coil of the primary coolant in the form of heated gas;

– through a special coil with an electric spiral built into it.

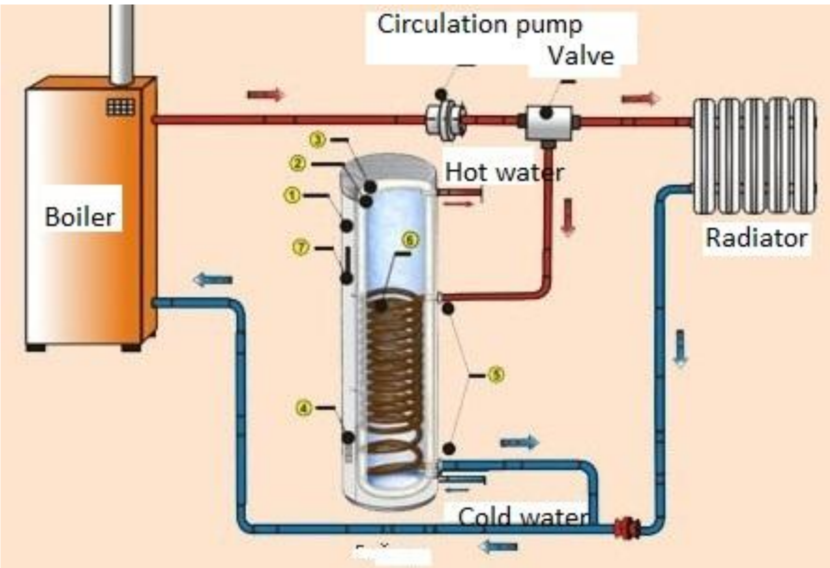

Figure 2. Scheme of heating and hot water supply in a private house
(1 – external housing; 2 – storage tank; 3 – heat-insulating coating;
4 – hole for cleaning the revision; 5 – water circulation pipe along the heating circuit;
6 – heat exchanger; 7 – thermometer)

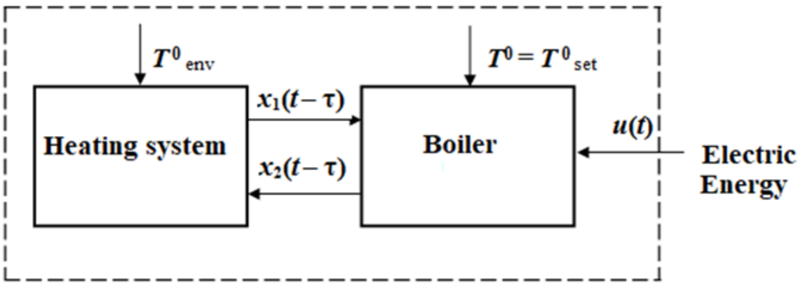

Figure 3. Structural scheme of the heat transfer process

In any case (with the exception of geothermal sources), bringing the heat supply system to a given temperature regime requires the consumption of energy resources. Therefore, optimization from the point of view of minimizing the consumption of energy resources in the transition mode is an important task.

It is known that the process of heat transfer is accompanied by the process of water mixing, i.e. recirculation process. Hence, this process can be described by a system of differential equations with a delayed argument, which is a simplified model of the Navier-

Stokes equations. At the same time, the amount of delay τ in our case can be determined based on the design parameters of the elements of the heating circuit and the nominal speed of water circulation. As a result, according to the structural scheme of the heating circuit (Fig.3), it is possible to write down the following system of differential-difference equations with the aftereffect:

$$\frac{dx_1(t)}{dt} = k_1[x_1(t) - T^0] + k_2[x_2(t-\tau) - x_1(t)],$$

$$\frac{dx_2(t)}{dt} = k_u u - k_3[x_2(t) - x_1(t-\tau)] \quad (11)$$

at the given boundary conditions $x_1(0) = T_{envir}; x_1(T) = x_2(T) = T^0$, where $x_1(t), x_2(t)$ – temperatures in the radiators and in the heating circuit coil.

The purpose of optimal control is to ensure the output of the temperature in the heating circuit at the set point at the final moment of time $t = T$ with minimal energy consumption T^0. For this formulation of the problem, the cost functional is defined by the expression:

$$I = [x_1(t) - T^0] + \int_0^T cu^2(t)dt. \quad (12)$$

We will also assume that the temperature T^0 at the time interval is equal to the ambient temperature T_{envir}, which corresponds to the real state of the heating system before operation. The solution of the optimal problem for the system (11) is very complicated and impossible in the analytical form due to the effect of the interaction. From the notation (11) let's move according to the dummy variable method to the notation of the form:

$$\frac{dx_1(t)}{dt} = k_1[x_1(t) - T^0] + k_2[z_2(t) - x_1(t)],$$

$$\frac{dx_2(t)}{dt} = k_u u - k_3[x_2(t) - z_1(t)], \quad (13)$$

$$\frac{dy_1(t,\theta)}{dt} + k_4\frac{dy_1(t,\theta)}{d\theta} = -k_5[y_1(t,\theta) - T^0],$$

$$\frac{dy_2(t,\theta)}{dt} + k_4\frac{dy_2(t,\theta)}{d\theta} = -k_5[y_2(t,\theta) - T^0]$$

under boundary conditions:

$$y_1(0,\theta) = 0;\ y_2(0,\theta) = 0;\ y_1(t,0) = x_1(t);\ y_2(t,0) = x_2(t), \quad (14)$$

where: $z_1(t) = y_1(t,\theta)$; $z_2(t) = y_2(t,\theta)$ – temperature in the pipes; u – the reduced control associated with the energy consumption to change the temperature of the coolants $(0 \le u_i \le u_{max})$; T^0 – the given temperature; k_i – a constant.

As an optimization method we shall use the principle of maximum [20].

For equations of system (13) and functional (12), let us compose the Hamiltonians:

$$H_\psi = \psi_0 cu^2(t) + \psi_1[-k_1(x_1 - T^0) + k_2(y_2(t,\tau) - x_1(t))] +$$

$$+ \psi_2[k_u u - k_3(x_2(t) - y_2(t,\tau)), \quad (15)$$

$$H_\varphi = -\varphi_1 k_5[y_1(t,\theta) - T^0) - \varphi_2 k_5[y_2(t,\theta) - T^0],$$

where ψ_i, φ_j are the auxiliary variables.

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In this case, the system of conjugate equations and boundary conditions have the form:

$$\frac{d\psi_0(t)}{dt} = 0,$$
$$\frac{d\psi_1(t)}{dt} = (k_1 + k_2)\psi_1(t) - \ k_3 \ \ _1(t, 0),$$
$$\frac{d\psi_2(t)}{dt} = \ k_3\psi_2(t) - \ k_5 \ \ _2(t, 0), \tag{16}$$
$$\psi_0(T) = -c;\ \psi_1(T) = -2(x_1(t) - T^0);\ \psi_2(t) = 0,$$

$$\frac{d\ \ _1(t)}{dx} + k_4 \frac{d\ \ _1(t)}{d\theta} = \ k_5 \ \ _1(t),$$
$$\frac{d\ \ _2(t)}{dx} + k_4 \frac{d\ \ _2(t)}{d\theta} = \ \ k_5 \ \ _2(t); \tag{17}$$
$$_1(T, \theta) = \frac{k_3}{k_4}\psi_2(t);\quad _2(T, \theta) = \frac{k_2}{k_u}\psi_2(t);$$
$$_1(T, \theta) = 0;\quad _2(T, \theta) = 0;\ t \in [0, T];\ \theta \in [0, \tau].$$

Since the optimal control must provide a maximum of the Hamiltonian, it follows from (15):

$$u^{\text{опт}}(t) = \begin{Bmatrix} 0\,, then\ \frac{k_u}{2c}\psi_2 < 0, \\ \frac{k_u}{2c}, then\ 0 < \frac{k_u}{2c} \le u_{max}, \\ u_{max}, then\ \frac{k_u}{2c} > u_{max}. \end{Bmatrix} \tag{18}$$

To determine the law of temperature change, it is necessary to solve together auxiliary equations (16), (17) and equations (13), (14) using the formula for optimal control (18). Joint solution of these equations is a solution of boundary value problem. In the general case, the solution of such problems requires application of methods of computational mathematics [21]. The optimal control algorithm is realized on the basis of the following iteration procedure:

Step 1. Set some initial value $x_1(T)$ and calculate $\psi_1(T) = 2 - 2(x_1(T) - T^0)$.

Step 2. Integrate the conjugate system of equations (16) in inverse time from $t = T$ to $t = 0$ and determine $\psi_2(t)$ and $\psi_1(t)$.

Step 3. From (18) determine $u(t)$.

Step 4. Substituting the found value $u(t)$into (11) and integrate this system from $t = 0$ to $t = T$, determine a new value of $x_1(T)$.

Step 5. If the found value $x_1(T)$ differs from the original value, go to step1. If they are close (to a given degree of accuracy), then the found control is optimal.

This algorithm can be used in the case of simultaneous output of heating circuits and hot water supply to stationary mode.

Stationary mode. It consists in the stabilization (maintenance) of the given temperature regime with small changes in the temperature of the external environment and the current volume of hot water consumption. We believe that the temperature of the primary heater is stable and corresponds to the specified temperature regimes. For the heating circuit, the main condition for the ability to control this mode is that the heating time in the coil of the heating circuit must be much shorter than the water circulation time in this circuit. In this case, the stabilization of the temperature regime is ensured with the help of a circulation pump by a corresponding change in the speed of water circulation. Stabilization of the temperature regime of the heating circuit is completely determined by the stability of the temperature of the accumulator tank, which, in turn, can be provided in the proposed heat exchanger by both constructive feedback and automatic compensation of water consumption. At the same time, the main condition is that the water heating time in the coil of the hot water supply circuit, determined from the dynamic model of the heat exchanger, should be much less than the water mixing time in the storage tank.

Conclusion

The problem of optimization of dynamic systems with an aftereffect, the mathematical model of which can be represented by a system of differential equations with a deviating argument, is considered. The dummy variable method proposed in this article allows applying the standard apparatus of optimization methods to the original system with an aftereffect by replacing the latter with an equivalent system of ordinary differential equations and partial differential equations in which there is no net delay. The method is generalized to systems with non-stationary delay and to systems with several processes having an aftereffect. The practical use of the dummy variable method is shown on the example of solving the problem of optimizing the heat exchange process in a private residential building, for which an optimal control algorithm has been synthesized.

REFERENCES

1. *Dolgiy Y. F.* Mathematical models of dynamical systems with reserve; textbook / Y. F. Dolgiy, P. G. Surkov. – Ekaterinburg: Urals State University Publisher, 2012. – 122 p.

2. *Prasolov A.V.* Dynamic models with time lag and their applications in economics and engineering. – Saint-Petersburg. Lan' Publisher, 2010. – 192 p.

3. *Elsgolz L.E., Norkin S.B.* Introduction to the theory of differential equations with a deviating argument. – M.: Nauka, 1971. – 296c.

4. *Yanushevsky R.T.* Control of objects with lagging. Moscow: Nauka, 1978. –410c

5. *Myshkis A.D.* Linear differential equations with delayed argument. – M.:Nauka, 1972. –351p.

6. *Turetsky X.* Analysis and synthesis of control systems with aftereffects. M.: Mashinostroenie, 1974. – 324p.

7. *Bellman R., Cook K.L.* Differential-differential equations. Moscow: The World, 1984. –548 p.

8. *Zabello JI.E.* A necessary condition of optimality of equality type for systems with lags. DU, 1999, vol.35, №10, pp.1452-1453.

9. *Dolgiy, Yu.F.* On Stability of Linear Dynamic Systems with Feedback. Mathematics, Mechanics, Informatics. Vestnik Permskogo Universiteta, vol.2(33),2016. – pp.74-81

10. *V. B. Kolmanovskii*, On the stability of some systems with arbitrary interaction, DAN of Russia, 1993, Vol. 331, 4, pp. 421-424.

11. *Yablonsky D.V.* Robust stabilizing control of hybrid stochastic systems with aftereffects // Studies in the field of natural sciences. 2014, № 9 [Electronic resource]. URL: https://science.snauka.ru/2014/09/8341.

12. *Pakshin P.V., Yablonskii D.V.* Robust stability and control of hybrid stochastic systems with time-delay // European Control Conference, ECC 1999 - Conference Proceedings, 1999 – pp. 2164-2169.

13. *A. Yakimenko*, Sufficient condition of stabilization for one system of neutral type. Proceedings of BSTU. Series3: Physico-mathematical sciences and informatics, №1,2019. – pp.5-8..

14. *Kharkovsky S.I.* Optimal stabilization of linear systems with neutral-type traces. - Research of mathematical models of technological systems of railway transport. – Samara: Samara Institute of Railway Transport Engineers, 1992, pp.8-16.

15. *Kostyuk V.I.,Stenin A.A., Ignatenko V.N.* Optimal fuel control of systems with delay. System Science, Poland, v.3, №2, 1977. – pp.159 -169.

16. *Stanin A.A., Burlakov V.M., Sukhodolsky K.V.* Synthesis of optimal laws of motion of dynamic systems with lagging. "Adaptive automatic control systems", No.3(20), 2000. – pp.25-29.

17. *A.Stenin, V.Pasko, I.Drozdovych* Designing of autonomous heat supply systems and optimization of energy resources at their exploitation/-Zaporizhia, National Technical University, №3(46), 2018. – pp.78-87.

18. *O. A. Ladyzhenskaya*, "The sixth problem of the millennium: the Navier-Stokes equations, existence and smoothness," Advances in Mathematical Sciences, 58:2(350), 2003. C. 45-78; Russian Mathematical Surveys, 58:2, 2003. –pp.251-286. DOI: https://doi.org/10.4213/rm610.

19. *Temam R.* Navier-Stokes equations. Theory and numerical analysis / R. Temam. – Moscow: Mir Publisher, 1981. – 408 c.

20. *Michael Athans.* Optimal Control: An Introduction to the Theory and Its Applications / Michael Athans, Peter L. Falb. - New York : McGraw-hill book company, 2006. – 879p.

21. *Feldman L.P., Petrenko A.I., Dmitrieva O.A.* Numerical Methods in Informatistics. K.: Publishing Group BHV, 2006. – 480p.

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