

## **CALIBRATION OF NAVIGATION PENDULOUS ACCELEROMETER BY TESTING ROTATING METHOD IN TERRESTRIAL GRAVITATIONAL FIELD**

*Abstract:* The metrological model of navigation pendulous accelerometer and a method of its statically calibration by testing rotating method in terrestrial gravitational field were considered. The results of calibration are presented for navigation pendulous accelerometer type AL-15.

*Keywords:* pendulous accelerometer, testing method, calibration.

### **Introduction**

Navigation pendulous accelerometers are the sensors of the primary information of practically all contemporary strapdown inertial navigation system (SINS) and orientation (SSO). It is known that the errors of the accelerometers produce errors in tasks solved by these systems.

By metrological model of pendulous accelerometers we understand the mathematical model of its output signal (real conversion function (CF) and the mathematical model of its error (MME), represented in the form of functional dependences on the measured projection of the apparent linear acceleration and reference ranges, which affect the pendulous accelerometers during a measurement. Individual certification of the CF and MME parameters is accomplished according to the results of the metrological testing of pendulous accelerometers.

Separate questions of CF and MME of navigation pendulous accelerometers constructions were studied in literature. As well were proposed methods of the experimental determination of the parameters of these models [1, 2]. One work [1] considers these tasks the most of all. In [2] the method of the test turnings of accelerometers in the Earth's gravitational field is proposed to be used to determine the CF and the MME of the parameters pendulous accelerometers. Nevertheless, only general recommendations were given, which do not determine the criterion of the selection of the necessary quantity of the pendulous accelerometers test regulations and do not lay claims to the testing equipment.

### **Problem statement**

The purpose of the report is to develop a metrological model of navigation pendulous accelerometers which would be sufficient for the practical usage and to describe the method of the experimental determination of the parameters of this model according to the results of its metrological testing.

## Metrological model of the navigation pendulous accelerometers

We accept sufficient for the practical usage of solving measuring tasks. As a basis we take requirements to create in Ukraine SINS and SSO, the results of their mathematical simulation and experimental research.

By metrological model of navigation pendulous accelerometer we understand the mathematical model of its output signal (real conversion function) and the mathematical model of its error, represented in the form of functional dependences on the measured projection of the apparent linear acceleration and references ranges, which affect the pendulous accelerometers during a measurement.

Let's accept that the sufficient for the practical usage during solving measuring tasks in the structure of these systems, real CF of the navigation pendulous accelerometers and its MME take the form:

– CF in the units of the output signal

$$Y_R = K_{0\Sigma} + K_1 a_i + K_2 a_i^2 + K_3 a_i^3 + M_O a_P + M_P a_O + M_i P a_i a_P; \quad (1)$$

– MME in the units of the measured apparent linear acceleration

$$\Delta a_i = \frac{Y_R - Y_I}{K_1} = k_{0\Sigma} + k_1 a_i + k_2 a_i^2 + k_3 a_i^3 + \delta_O a_P + \delta_P a_O, \quad (2)$$

here  $Y_R$  – the real individual CF of an accelerometer;  $Y_I = K_{10} a_i$  – its ideal (without errors) linear CF,  $K_{10}, K_1$  – the ideal and the real pendulous accelerometers conversion factor,  $a_i$  – the measured projection of the apparent acceleration on the input axis (IA) of the accelerometer;  $a_O, a_P$  – the immeasurable projections of the apparent acceleration on its output axis (OA) and pent axis (PA) accordingly;  $K_{0\Sigma}, k_{0\Sigma}$  – according to bias of CF and additive errors of bias;  $k_1$  – multiplicative errors of CF;  $K_2, k_2, K_3, k_3$  – the systematic nonlinearity factor of CF and MME;  $M_P, \delta_P, M_O, \delta_O$  – the systematic cross sensitivity factor and the base plane errors (as small angle of the nonorthogonality of axis IA relative to axes PA and OA accordingly).

Let's consider the components of MME (2). As a basis we take the fact that navigation pendulous accelerometers has a large dynamic range ( $\approx 10^6 \dots 10^7$ ) and is intended for the measurement of both very small from ( $2 \dots 5 \cdot 10^{-5} g$ ) and high to accelerations with an assigned accuracy (error). The small accelerations are relative to the pendulous accelerometers threshold of response in SINS initial orientation, navigation during the cruising motion of an object, and the determination of SSO small orientation angles ( $to 1 \dots 20 g$ ). The high accelerations are close to the pendulous accelerometers upper limit of measurement in navigation during the maneuverable motion of an object, the determination of SSO large orientation angles. In this case all SINS and SSO algorithmically compensated errors are considered to be systematic, no compensable errors – random.

**Bias errors (BE)** pendulous accelerometers  $K_{0\Sigma}$  – let us determine errors in the bias pendulous accelerometers as follows:

$$K_{0\Sigma} = \hat{k}_0 \pm \mathring{k}_0(t) \pm \mathring{P} \pm \mathring{h},$$

where  $\hat{k}_0$  – BE systematic value;  $\mathring{k}_0(t)$  – the random bias error;  $\mathring{h}, \mathring{p}$  – according to the random bias errors caused by hysteresis (mainly temperature hysteresis  $\mathring{h}_T$ ) and threshold sensitivity (it is determined by the level of electronics noise in the output signal of accelerometer). The temperature dependence of bias systematic value is determined by the expressions:

$$\mathring{k}_0(\Delta T) = \hat{k}_{0N} + \lambda_{T1}\Delta T, \text{ at } \Delta T \succ 0;$$

$$\mathring{k}_0(\Delta T) = \hat{k}_{0N} + \lambda_{T2}\Delta T, \text{ at } \Delta T \prec 0,$$

where  $\hat{k}_{0N}$  – the certifying systematic value of bias in standard conditions (temperature  $T_0$  of the accelerometer calibration);  $\lambda_{T1}, \lambda_{T2}$  – the certifying temperature bias factor (bias temperature coefficient);  $\Delta T = T_{current} - T_0$  – a change of the temperature,  $T_{current}$  – the current ambient temperature.

The most essential sources of the random errors in bias  $\mathring{k}_0(t)$  are no repeatability from one launch to the next one (composite repeatability)  $\mathring{k}_{0L}$ , its drift  $\mathring{k}_{0D}$  and low-frequency fluctuations  $\mathring{k}_{0F}$  (deviation from the drift line) during the launch. These components have a weak correlation; therefore an expression, which determines the random bias error, can be recorded in the form:

$$\mathring{k}_0(t) \approx \pm 0,5\sqrt{\mathring{k}_{0L}^2 + \mathring{k}_{0D}^2(t) + \mathring{k}_{0F}^2}, \quad (3)$$

where,  $\mathring{k}_{0L} = \pm t_P\sigma_L$ ,  $\mathring{k}_{0F} = \pm t_P\sigma_F$ ,  $t_P$  – the student factor, whose value is taken in the range from 2.7 to 3;  $\sigma_L, \sigma_F$  – the corresponding mean-square deviation (MSD) of bias no repeatability from one launch to the next one and the bias fluctuations during a launch of an accelerometer (composite repeatability);  $\mathring{k}_{0D}(t) = \mathring{\gamma}_D t$ ,  $\mathring{\gamma}_D$  – bias factor,  $t$  – time.

**Scale factor errors (SFE).** Let's write down the temperature model of SFE in a form:

$$\mathring{k}_{1T}(\Delta T) = \beta_{T1}\Delta T, \text{ at } \Delta T \succ 0;$$

$$\mathring{k}_{1T}(\Delta T) = \beta_{T2}\Delta T, \text{ at } \Delta T \prec 0,$$

where  $\beta_{T1}, \beta_{T2}$  – SC certified temperature factors.

Let's determine random scale factor errors  $\mathring{k}_1$  as its instability in a form

$$\mathring{k}_1 = \pm t_P\sigma_{FC},$$

where SF MSD of an average value.

**Base plane systematic errors**  $\delta_O, \delta_P$  depend as well on a change of the ambient temperature:

$$\delta_{O(P)}(\Delta T) = \delta_{O(P)N} + \gamma_{T1}\Delta T, \text{ at } \Delta T \succ 0;$$

$$\delta_{O(P)}(\Delta T) = \delta_{O(P)N} + \gamma_{T2}\Delta T, \text{ at } \Delta T \prec 0,$$

where  $\delta_{O(P)N}$  – an error in the base place in standard conditions;  $\gamma_{T1}, \gamma_{T2}$  – the certifying base plane temperature factor errors. In this way, a convenient for the practical usage model of navigation pendulous accelerometers error can be represented in the form:

$$\begin{aligned} \Delta a_i(a_i, \Delta T, t) \approx & \hat{k}_{0N} + \lambda_{T1(2)}\Delta T \pm (0,5\sqrt{\hat{k}_{0L}^2 + \hat{k}_{0D}^2(t) + \hat{k}_{0F}^2} + \dot{h}_T + \dot{p}) + \\ & + (\beta_{T1(2)}\Delta T \pm \hat{k}_1)a_i + k_2a_i^2 + k_3a_i^3 + (\delta_o + \gamma_{T1(2)}\Delta T)a_p + \\ & + (\delta_p + \gamma_{T1(2)}\Delta T)a_o + \delta_{ip}a_ia_p. \end{aligned} \tag{4}$$

### A metrological testing method and its results

In the article was considered the method of metrological test of navigation pendulous accelerometers in order to create an experimental determination of its error model (4). Was used the method of test turning in the Earth’s gravitation field. As well were laid claims to the testing equipment. The method was tested during the conducting of metrological tests of the developed in Ukraine navigation compensating pendulous accelerometers of the ALO type (AL-15). These accelerometers have the following standard metrological characteristics [2]: bias – is less than  $5mg$ ; the bias no repeatability – is not more than  $50mkg$ ; drift in the launch (stability) – is not more than  $50mkg/6h$ ; the bias temperature sensitivity – is not more than  $50mkg/^\circ C$ ; scale factor  $-1, 2 \dots 1, 5mA/g$ ; scale factor stability – is not more than  $100ppm$ ; scale factor temperature sensitivity – is not more than  $500ppm/^\circ C$ ; the error of the base plane – is not more than  $2000mkrad$ ; operating temperatures range –  $(-40 \dots + 80)^\circ C$ .

On Fig. 1 the stand for the navigation PA metrological tests is down.

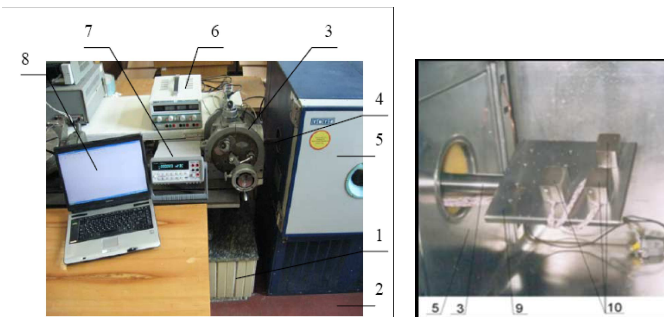


Figure 1 – The general kind of stand for the metrological pendulous accelerometers tests

The components of a stand are the following basic metrological equipments: foundation 1, untied from a construction 2, with daily variations of

base, which are not more than one angular second  $\pm 1 \dots 2$ ; the acceleration setting device 3 – optical index head OIH-5E, which provides an error of the acceleration task during the test turnings method not more than  $\pm 2 \cdot 10^{-5} g$ ; the bubble level is for the adjusting plate installation of shaft OIH 4 into the horizontal plane of the horizon with an error of one are second  $\pm 2;5$  – type of heat chamber TWT-2, an error of the assigned temperature measurement  $\pm 0,1^\circ C$ ; a precision multichannel voltmeter 7, type Agilent 33401A, the class of the voltmeter precision is 0,003% of the sub-band.

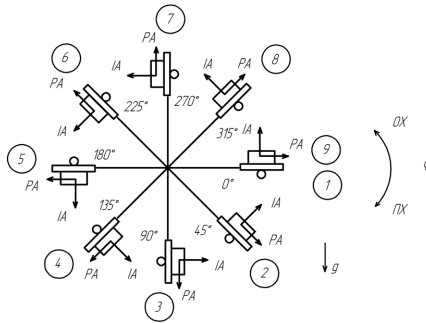


Figure 2 – The test turnings diagram during the pendulous accelerometers

The method of test turnings in the Earth’s gravitation field was used for pendulous accelerometers calibration. It was carried, as a minimum, at 3 different temperatures (at normal temperature, and also at minimum and maximum temperatures from the range of pendulous accelerometers operating temperatures). It was shown that the usage of the 8-points test is the most expedient. During the test, 3 times were reached the pendulous accelerometers test positions, which are relative to the local horizon plane of the forward stroke (FS) and the back stroke (BS). As a result were obtained three tables of the accelerometer output signals  $Y_{JK}^T$ , where  $J = 1, 2, \dots 9$  – the number of the test position according to the Fig.2;  $k = 1, 2, \dots 6$  – the number of the motion of measurement (FS,BS) in three cycles of the measurement ( $k = 1, 3, 5$  for FS and  $k = 2, 4, 6$  for BS);  $T$  – the value of the temperature, at which the calibration took place. At the place of calibration pendulous accelerometer  $g = 9, 8106m \cdot c^{-2}$  and the expressions of output signals is:

$$Y_1 = K_{0\Sigma} + K_1 g(1 - 0,5\delta_0^2) + K_2 g^2(1 - \delta_0^2) + K_3 g^3(1 - 1,5\delta_0^2) + M_{o(p)}g(\delta_0);$$

$$Y_2 = K_{0\Sigma} + K_1 \frac{g}{\sqrt{2}}(1 - \delta_0) + K_2 \frac{g^2}{2}(1 - 2\delta_0) + K_3 \frac{g^3}{2\sqrt{2}}(1 - 3\delta_0) + M_{o(p)} \frac{g}{\sqrt{2}}(1 - \delta_0) + M_{ip} \frac{g}{2}(1 - \sqrt{2}\delta_0);$$

$$Y_3 = K_{0\Sigma} - K_1 g\delta_0 + K_2 g^2 \delta_0^2 - K_3 g^3 \delta_0^3 + M_{o(p)}g(1 - 0,5\delta_0^2) - M_{ip}g\delta_0;$$

$$Y_4 = K_{0\Sigma} - K_1 \frac{g}{\sqrt{2}}(1 + \delta_0) + K_2 \frac{g^2}{2}(1 + 2\delta_0) - K_3 \frac{g^3}{2\sqrt{2}}(1 + 3\delta_0) + M_{o(p)} \frac{g}{\sqrt{2}}(1 + \delta_0) + M_{ip} \frac{g}{2}(-1 - \sqrt{2}\delta_0); \quad (5)$$

$$Y_5 = K_{0\Sigma} - K_1 g(1 - 0, 5\delta_0^2) + K_2 g^2(1 - \delta_0^2) - K_3 g^3(1 - 1, 5\delta_0^2) - M_{o(p)} g(\delta_0);$$

$$Y_6 = K_{0\Sigma} - K_1 \frac{g}{\sqrt{2}}(1 - \delta_0) + K_2 \frac{g^2}{2}(1 - 2\delta_0) - K_3 \frac{g^3}{2\sqrt{2}}(1 - 3\delta_0) - M_{o(p)} \frac{g}{\sqrt{2}}(1 - \delta_0) + M_{ip} \frac{g}{2}(1 + \sqrt{2}\delta_0);$$

$$Y_7 = K_{0\Sigma} + K_1 g\delta_0 + K_2 g^2\delta_0^2 + K_3 g^3\delta_0^3 - M_{o(p)} g(1 - 0, 5\delta_0^2) + M_{ip} g\delta_0;$$

$$Y_8 = K_{0\Sigma} + K_1 \frac{g}{\sqrt{2}}(1 + \delta_0) + K_2 \frac{g^2}{2}(1 + 2\delta_0) + K_3 \frac{g^3}{2\sqrt{2}}(1 + 3\delta_0) - M_{o(p)} \frac{g}{\sqrt{2}}(1 + \delta_0) + M_{ip} \frac{g}{2}(-1 + \sqrt{2}\delta_0).$$

The searched CF and MME coefficients are connected to the output PA signals in the test calibration positions by the expressions (5):

$$\hat{K}_{0\Sigma} = 0, 5(Y_3 + Y_7);$$

$$\hat{K}_1 \approx K_1(1 - 0, 5\delta_0^2) = \frac{1}{8g} [Y_1 + Y_9 - 2Y_5 - \sqrt{2}(Y_2 + Y_8 - Y_4 - Y_6)];$$

$$\hat{K}_2 = \frac{1}{2g^2} [Y_2 + Y_4 + Y_6 + Y_8 - 4\hat{K}_{0\Sigma}]; \hat{K}_3 = \frac{\sqrt{2}}{g^3} [Y_2 + Y_6 - \sqrt{2}g\hat{K}_1]; \quad (6)$$

$$\hat{M}_{o(p)} = \frac{0, 5}{g}(Y_7 - Y_3); \hat{M}_{ip} = \frac{1}{2g} [Y_2 + Y_6 - Y_4 - Y_8] + 2K_2 g\delta_0;$$

$$\hat{k}_{0\Sigma} = \hat{K}_{0\Sigma} \hat{K}_1^{-1}; \hat{\delta}_{o(p)} = \hat{M}_{o(p)} \hat{K}_1^{-1}; \hat{k}_2 = \hat{K}_2 \hat{K}_1^{-1}; \hat{k}_3 = \hat{K}_3 \hat{K}_1^{-1}; \hat{\delta}_{ip} = \hat{M}_{ip} \hat{K}_1^{-1}.$$

On Figure 3 are given standard pendulous accelerometers temperature dependences  $\hat{k}_0(\Delta T)$  and  $K_1(\Delta T)$ . On the its operating temperature range  $\hat{h}_T$ .

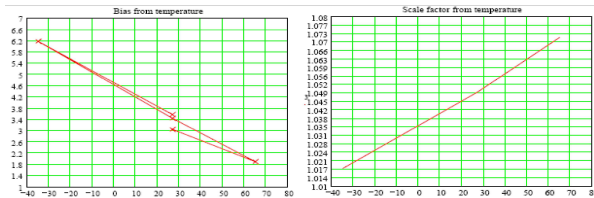


Figure 3 – Temperature hysteresis of the accelerometer bias  $\hat{k}_0(\Delta T)$ (a) and temperature dependents pendulous accelerometers  $K_1(\Delta T)$

The estimations model of the systematic factors errors of the pendulous accelerometers:

$$\hat{k}_{0\Sigma} = 3,324mg; \hat{k}_1 = 15ppm(3\sigma); \hat{k}_2 = 105\mu g/g^2; \hat{k}_3 = 87\mu g/g^3;$$

$$\hat{\delta}_0 = 1,15mrad \approx \hat{\delta}_p; \lambda_{T1} = -3,9 \cdot 10^{-5}g/^{\circ}C; \lambda_{T2} = -4,5 \cdot 10^{-5}g/^{\circ}C;$$

$$\beta_{T1} = 520ppm/^{\circ}C; \beta_{T2} = 480ppm/^{\circ}C; \hat{h} \approx \hat{h}_T = \pm 3 \cdot 10^{-4}g(3\sigma).$$

Estimations of the components of the random bias error,  $\hat{k}(t)$ , ( $3\sigma$ ) (fig. 4):  
 $\hat{k}_{0L} \approx \pm 2.10^{-5}g, t \approx 10 \text{ min}; \hat{k}_{0D}(t) \approx \hat{k}_{0F} \leq \pm 2.10^{-5}g$  for 12 hours.

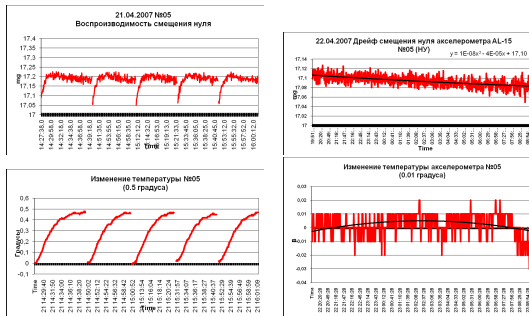


Figure 4 – The bias (a,c) and temperature changes (b,d) output signals  $\hat{k}_0(t)$

## Conclusions

A simple metrological model of navigation pendulous accelerometers is represented; the experimental model determination of the factors model according to the results of its metrological tests by the test turnings in the Earth’s gravitation field model is described. It is shown that the most expedient is the 8- points test, with the threefold progress test positions of the pendulous accelerometers relativity to the local horizontal plane. Requirements to the test equipment are started, a stand for the metrological navigation pendulous accelerometers tests is described, the results of the factors determining metrological accelerometer model are given.

## Bibliography

1. IEEE Standard Specification Format Guide and Test procedure for Linear, Single-Axis, Nongyroscopic Accelerometers/IEEE std 1293,1998.
2. Zbrytsky A.V , Chernyak N.G. , Skripkovskiy G.A. Creation of Low Cost Linear Accelerometers for Navigation and Control Systems// Symposium Gyro Technology. -Stuttgart, Germany, 2005. – P.4.1 – 4.11.

Отримано 10.12.2009 р.